MATHEMATICS KNOWLEDGE REVIEW

The Mathematics Knowledge subtest of the ASVAB deals with the ability to use basic mathematical relationships learned in math courses, such as algebra, geometry, and trigonometry. This subtest tests your knowledge of math principles, concepts, and procedures. This review section can also be considered a continuation of the Arithmetic Reasoning Review given previously. Reread that section as background material for Mathematics Knowledge Review.

Adding Fractions

With the Same Denominator

Fractions with the same denominator are added directly, as each part represents a part of the same value. Add the numerators and place the sum over the common denominator. If necessary, simplify to the simplest form.

$$\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$$

$$\frac{1}{5} + \frac{3}{5} + \frac{4}{5} = \frac{8}{5} = 1\frac{3}{5}$$

With Different Denominators

Fractions with different denominators may not be added directly because parts of different values are involved. They must be renamed as equivalent fractions having the same common denominator. After all the fractions have the same denominator, add the numerators and place the total over the common denominator. If necessary, simplify to the simplest form.

$$\frac{1}{2}+\frac{1}{4};\ \frac{1}{2}+\frac{1}{3}+\frac{3}{4}$$

$$\begin{array}{r} \frac{1}{2} \\ +\frac{1}{4} \\ \hline \end{array} \qquad \begin{array}{r} \frac{2}{4} \\ +\frac{1}{4} \\ \hline \frac{3}{4} \end{array} \qquad \begin{array}{r} \frac{1}{2} \\ \frac{1}{3} \\ +\frac{3}{4} \\ \hline \end{array} \qquad \begin{array}{r} \frac{6}{12} \\ \frac{4}{12} \\ +\frac{9}{12} \\ \hline \frac{19}{12}=1\frac{7}{12} \end{array}$$

Adding Mixed Numbers

In adding mixed numbers, first add all the whole numbers, then add all fractions, and then add the sum of the whole numbers to the sum of the fractions.

$$4\frac{1}{4}+3\frac{1}{2}+2\frac{1}{2}$$

$$\begin{array}{r} 4\frac{1}{4} \\ 3\frac{1}{2} \\ +2\frac{1}{2} \\ \hline \end{array} \qquad \begin{array}{r} 4\frac{1}{4} \\ 3\frac{2}{4} \\ +2\frac{2}{4} \\ \hline \end{array}$$

$$9+\frac{5}{4}=9+1\frac{1}{4}=10\frac{1}{4}$$

Adding Percents

As percents are actually fractions with 100 as the same common denominator, they may be added directly.

$$\begin{array}{r} 6\% \\ 4\% \\ +9\% \\ \hline 19\% \end{array} \qquad \begin{array}{r} 8\% \\ +17\% \\ \hline 25\% \end{array} \qquad \begin{array}{r} 20\% \\ 25\% \\ +35\% \\ \hline 80\% \end{array}$$

If fractional parts of a percent are involved in the addition, the addends may be added as decimals or added directly after the fractional parts are renamed with the same common denominator.

$$15\frac{1}{2}\% + 8\frac{1}{4}\%$$

$$\begin{array}{r} 0.155 \\ +0.0825 \\ \hline 0.2375 \end{array} \qquad \begin{array}{rr} 15\frac{1}{2}\% & 15\frac{2}{4}\% \\ +\,8\frac{1}{4}\% & +\,8\frac{1}{4}\% \\ \hline & 23\frac{3}{4}\% \end{array}$$

Subtracting Fractions

With the Same Denominator

Fractions with the same denominator may be subtracted directly.

$$\frac{3}{5} - \frac{2}{5} = \frac{1}{5} \qquad \frac{7}{8} - \frac{1}{8} = \frac{6}{8}, \text{ which simplifies to } \frac{3}{4}.$$

With Different Denominators

Fractions with different denominators are not subtracted directly because parts of different values are involved. They must be renamed as equivalent fractions having the same common denominator. After the fractions have the same denominator, subtract the numerators and place the remainder over the common denominator.

$$\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12} \qquad \frac{4}{5} - \frac{1}{2} = \frac{8}{10} - \frac{5}{10} = \frac{3}{10}$$

Subtracting Mixed Numbers

With the Same Denominator

If mixed numbers have the same denominator, rename them as improper fractions and subtract directly.

$$4\frac{1}{3} - 2\frac{2}{3} = \frac{13}{3} - \frac{8}{3} = \frac{5}{3} = 1\frac{2}{3}$$

With Different Denominators

If mixed numbers have different denominators, first rename as improper fractions, then rename as equivalent fractions with the same common denominator and subtract directly.

$$3\frac{1}{3} - 2\frac{3}{4} = \frac{10}{3} - \frac{11}{4} = \frac{40}{12} - \frac{33}{12} = \frac{7}{12}$$

Subtracting Percents

Percents are fractions with 100 as the same common denominator and may be subtracted directly.

$$70\% \text{ minus } 30\% = 40\%$$

If fractional parts of a percent are involved in the subtraction, rename them as decimals or rename the fractional parts with the same common denominator.

$$8\frac{1}{4}\% - 5\frac{2}{5}\%$$

$$\begin{array}{r} 0.0825 \\ -\ 0.0540 \\ \hline 0.0285 \end{array} \quad \begin{array}{r} 8\frac{1}{4}\% \\ -\ 5\frac{2}{5}\% \\ \hline \end{array} \quad \begin{array}{r} 8\frac{5}{20}\% \\ -\ 5\frac{8}{20}\% \\ \hline \end{array} \quad \begin{array}{r} 7\frac{25}{20}\% \\ -\ 5\frac{8}{20}\% \\ \hline 2\frac{17}{20}\% \end{array}$$

Multiplying Fractions

With fractions, the product of the numerators divided by the product of the denominators gives the final answer or product.

$$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}, \text{ which simplifies to } \frac{1}{3}.$$

Dividing a number in the numerator by the same number in the denominator simplifies the computation.

$$\frac{1}{\cancel{2}_1} \times \frac{\cancel{2}^1}{3} = \frac{1}{3}$$

Dividing a common factor is particularly useful when multiplying many fractions.

$$\frac{3}{5}\times\frac{1}{2}\times\frac{2}{3}\times\frac{5}{8}=\frac{3\times1\times2\times5}{5\times2\times3\times8}=\frac{30}{240}=\frac{1}{8}$$

With dividing common factors:

$$\frac{3}{5}\times\frac{1}{2}\times\frac{2}{3}\times\frac{5}{8}=\frac{\overset{1}{\cancel{3}}\times1\times\overset{1}{\cancel{2}}\times\overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}}\times\underset{1}{\cancel{2}}\times\underset{1}{\cancel{3}}\times8}=\frac{1}{8}$$

Note that dividing common factors is permitted when only multiplication is involved.

When multiplying fractions and whole numbers, use the same procedure as when multiplying fractions only. Whole numbers are basically fractions with the whole number as the numerator and one as the denominator.

$$4=\frac{4}{1}\qquad 10=\frac{10}{1}\qquad 150=\frac{150}{1}$$

When multiplying a fraction and a whole number, the word *of* means *multiply* by.

$\frac{1}{2}$ of 48 means $\frac{1}{2}\times\frac{48}{1}$, which equals $\frac{48}{2}$ and equals 24.

Multiplying Mixed Numbers

There are several methods that may be used in multiplying mixed numbers.

When the numbers are small valued, rename the mixed numbers as improper fractions and then multiply in the usual manner.

$$3\frac{1}{4}\times16=\frac{13}{4}\times\frac{16}{1}=\frac{208}{4}=52$$

$$2\frac{2}{3}\times1\frac{1}{4}=\frac{8}{3}\times\frac{5}{4}=\frac{40}{12}=\frac{10}{3}=3\frac{1}{3}$$

$$1\frac{1}{2}\times2\frac{2}{3}\times3\frac{3}{4}=\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}}\times\frac{\overset{1}{\cancel{8}}}{\underset{1}{\cancel{3}}}\times\frac{15}{\underset{1}{\cancel{4}}}=15$$

When the numbers are great and the fractional parts have exact decimal equivalents, rename the fractional parts as decimals and then multiply.

$$342\frac{1}{4} \times 609\frac{3}{4} = 342.25 \times 609.75$$

$$\begin{array}{r} 342.25 \\ \underline{\times 609.75} \\ 171125 \\ 239575 \\ 308025 \\ \underline{2053500} \\ 208,686.9375 \end{array}$$

When the numbers are not small valued and the fractional parts have no exact decimal equivalents, use the partial product method as follows:

$$386\frac{3}{7} \times 245\frac{1}{3}$$

$$\begin{array}{rl} 386\frac{3}{7} & \\ \underline{\times 245\frac{1}{3}} & \\ \frac{3}{21} & \left(\frac{1}{3} \times \frac{3}{7}\right) \\ 128\frac{2}{3} & \left(\frac{1}{3} \times 386\right) \\ 105 & \left(\frac{3}{7} \times 245\right) \\ \underline{94,570} & (245 \times 386) \\ 94,803\frac{17}{21} & \end{array}$$

Find the partial products of the

a. fractional parts of the multiplier and the multiplicand.
b. fractional part of the multiplier and the whole number of the multiplicand.
c. fractional part of the multiplicand and the whole number of the multiplier.
d. whole number part of the multiplier and the whole number part of the multiplicand.

Add the partial products and, if necessary, simplify the fractional part of the answer.

Multiplying Percents

Since a percent is actually a fraction with a denominator of 100, it may be multiplied after renaming the percent as a decimal or as a fraction.

$$33\% \times 8\% = .33 \times .08 = .0264 = 2\frac{64}{100}\% = 2\frac{32}{50}\% = 2\frac{16}{25}\%$$

$$75\% \times 50\% = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8} = 37\frac{1}{2}\%$$

Other Multiplication Properties

1. If the multiplier and the multiplicand are interchanged, the product will remain the same.

   $5 \times 4 = 20 \qquad 4 \times 5 = 20$

2. If the numbers being multiplied are associated in different ways, the product will remain the same.

   $3 \times (5 \times 2) = 2 \times (3 \times 5) = 5 \times (2 \times 3)$

3. Multiplying any number by 1 does not change the number.

   $8 \times 1 = 8 \qquad 1.5 \times 1 = 1.5 \qquad \frac{3}{4} \times 1 = \frac{3}{4}$

4. Zero times any number equals zero.

   $8 \times 0 = 0 \qquad 1.5 \times 0 = 0 \qquad \frac{3}{4} \times 0 = 0$

5. a. To multiply by 10, move the decimal point in the number one place to the right:

   $1.36 \times 10 = 13.6, 13.6 \times 10 = 136, 136 \times 10 = 1{,}360$

   b. To multiply by 100, move the decimal point in the number two places to the right.

   $3.61 \times 100 = 361, 36.1 \times 100 = 3{,}610, 361 \times 100 = 36{,}100$

   c. To multiply by 1,000, move the decimal point in the number three places to the right.

$4.875 \times 1{,}000 = 4{,}875$

$48.75 \times 1{,}000 = 48{,}750$

$487.5 \times 1{,}000 = 487{,}500$

$4{,}875 \times 1{,}000 = 4{,}875{,}000$

When multiplying by 10, 100, 1,000, etc., move the decimal point in the number as many places to the right as there are zeros in the multiplier. If necessary, add zero(s) to the product.

Dividing Fractions

With fractions, multiply by the reciprocal of the divisor.

$\frac{3}{4} \div \frac{1}{2}$ is rewritten as $\frac{3}{4} \times \frac{2}{1}$, which equals $\frac{6}{4} = 1\frac{1}{2}$

$\frac{2}{3} \div \frac{3}{4}$ is rewritten as $\frac{2}{3} \times \frac{4}{3}$, which equals $\frac{8}{9}$

$3 \div \frac{3}{4}$ is rewritten as $\frac{3}{1} \times \frac{4}{3}$, which equals $\frac{12}{3} = 4$

$\frac{1}{4} \div 2$ is rewritten as $\frac{1}{4} \times \frac{1}{2}$, which equals $\frac{1}{8}$

Dividing Percents

A percent divided by a percent is similar to dividing two fractions with 100 as the common denominator.

$25\% \div 50\%$ is rewritten as $\frac{25}{100} \times \frac{100}{50}$, which equals $\frac{25}{50} = \frac{1}{2}$

$40\% \div 40\%$ is rewritten as $\frac{40}{100} \times \frac{100}{40}$, which equals 1

Note that when a percent is divided by a percent, the result is a whole number or a fraction.

$\frac{1}{2} \div 25\%$ is rewritten as $\frac{1}{2} \times \frac{100}{25}$, which equals $\frac{100}{50} = 2$

$25\% \div \frac{1}{2}$ is rewritten as $\frac{25}{100} \times \frac{2}{1}$, which equals $\frac{50}{100} = \frac{1}{2}$

Similarly, when a fraction is divided by a percent or a percent is divided by a fraction, the result is also a whole number or a fraction.

Percents may also be renamed to decimal form before division.

$25\% \div 50\%$ is rewritten as $\frac{.25}{.50}$, which equals $\frac{25}{50} = \frac{1}{2}$

$\frac{1}{2} \div 25\%$ is rewritten as $\frac{1}{2} \div .25 = \frac{1}{2} \times \frac{1}{.25} = \frac{1}{.50} = \frac{100}{50} = 2$

$25\% \div \frac{1}{2}$ is rewritten as $.25 \div \frac{1}{2} = .25 \times 2 = .50 = \frac{1}{2}$

Dividing Mixed Numbers

There are several methods that are used to divide mixed numbers.

When the numbers are small valued, rename the mixed numbers as improper fractions and then divide in the usual manner.

$2\frac{1}{4} \div 1\frac{1}{2}$ is rewritten as $\frac{9}{4} \div \frac{3}{2}$, which becomes $\frac{9}{4} \times \frac{2}{3}$ and equals $\frac{3}{2} = 1\frac{1}{2}$

$1\frac{1}{2} \div 2\frac{1}{4}$ is rewritten as $\frac{3}{2} \div \frac{9}{4}$, which becomes $\frac{3}{2} \times \frac{4}{9}$ and equals $\frac{2}{3}$

When the mixed numbers are great and the fractional parts have exact decimal equivalents, rename the fractional parts as decimals and then divide.

$432\frac{3}{5}$ divided by $156\frac{1}{2}$ is rewritten as $432.6 \div 156.5$

$$1565\overline{)4326}$$

When the mixed numbers are great and the fractional parts do not have exact decimal equivalents,

1. Multiply both the dividend and the divisor by the denominator of the fraction if only one fraction is involved:

$$475 \div 28\frac{2}{3} = \frac{475 \times 3}{28\frac{2}{3} \times 3} = \frac{475 \times 3}{\frac{86}{3} \times 3} = \frac{1425}{86}$$

$$27\frac{1}{6} \div 39 = \frac{27\frac{1}{6} \times 6}{39 \times 6} = \frac{\frac{163}{6} \times 6}{39 \times 6} = \frac{163}{234}$$

2. Multiply both the dividend and the divisor by the least common denominator if two fractions are involved:

$$42\frac{2}{3} \div 12\frac{1}{6} = \frac{42\frac{2}{3} \times 6}{12\frac{1}{6} \times 6} = \frac{\frac{128}{3} \times 6}{\frac{73}{6} \times 6} = \frac{256}{73}$$

Other Division Properties

1. If the division is not exact, the number that is left is the remainder. This remainder becomes the numerator, and the divisor becomes the denominator of this common fraction that is added to the quotient:

$$4\overline{)1207}$$

$$301\frac{3}{4}$$

2. The remainder may also be shown as a decimal. The decimal point is placed to the right of the unit digit of the dividend and zero digits are added. Divide to the desired number of decimal places:

$$4\overline{)1207}.00$$

$$301.75$$

3. Dividing any number by 1 does not change the number:

$$25 \div 1 = 25 \qquad 4.7 \div 1 = 4.7 \qquad \frac{3}{5} \div 1 = \frac{3}{5}$$

4. Dividing by zero is not permissible, because the answer indicated would be undefined.

5. a. To divide by ten, move the decimal point in the number one place to the left:

$$\frac{136}{10} = 13.6 \qquad \frac{13.6}{10} = 1.36 \qquad \frac{1.36}{10} = .136$$

b. To divide by 100, move the decimal point in the number two places to the left:

$$\frac{350}{100} = 3.50 \qquad \frac{35}{100} = 0.35 \qquad \frac{3.5}{100} = .035$$

c. To divide by 1000, move the decimal point in the number three places to the left:

$$\frac{4055}{1000} = 4.055 \qquad \frac{40.55}{1000} = .04055$$

$$\frac{405.5}{1000} = .4055 \qquad \frac{4.055}{1000} = .004055$$

When dividing by 10, 100, 1000, etc., move the decimal point in the number as many places to the left as there are zeros in the divisor. If necessary, use zeros at the left of the dividend.

Factors of a Product

When two or more numbers are multiplied to produce a certain product, each number is known as a *factor* of the product.

$1 \times 8 = 8$ (1 and 8 are factors of the product)

$2 \times 4 = 8$ (2 and 4 are factors of the product)

Base

A *base* is a number used as a factor two or more times. $2 \times 2 \times 2$ may be written 2^3, which is read "2 cubed" or "2 to the third power." In the equation $2^3 = 8$, 2 is called the base.

Exponent

The *exponent* is the number that shows how many times the base is to be used as a factor. 10^2 is a short way of writing 10 ′ 10. 10 is called the base in 10^2; 2 is called the exponent.

$a^4 = a \times a \times a \times a$ (a is the base, 4 is the exponent)

$5^3 = 5 \times 5 \times 5$ (5 is the base, 3 is the exponent)

Power

Power is an expression such as 3^2. 3^2 is the second power of three (3 ′ 3) and is equal to 9. 2^4 is the fourth power of two $(2 \times 2 \times 2 \times 2)$ and is equal to 16. Note that all the factors of the product are equal.

Reciprocal

If the product of two numbers is 1, either number is called the *reciprocal* of the other number. 4 is the reciprocal of $\frac{1}{4}$; $\frac{1}{4}$ is the reciprocal of 4; $4 \times \frac{1}{4} = 1$. Similarly, $\frac{3}{5}$ is the reciprocal of $\frac{5}{3}$; $\frac{5}{3}$ is the reciprocal of $\frac{3}{5}$; $\frac{3}{5} \times \frac{5}{3} = 1$.

Factorial

The *factorial* of a natural or counting number is the product of that number and all the natural numbers less than it. **4 factorial**, written as $4! = 4 \times 3 \times 2 \times 1 = 24$.

Prime Number

A *prime number* is a natural or counting number with exactly two factors, namely itself and 1. Examples of prime numbers are 2, 3, 5, 7, 11, 13, 17, etc.

Roots

Square Root

The *square root* of a number is a number that, when raised to the second power, produces the given number. For example, the square root of 16 is 4 because $4^2 = 16$. $\sqrt{\ }$ is the symbol for square root.

The square roots of the most common perfect squares are given in the following table below.

NUMBER	PERFECT SQUARE	NUMBER	PERFECT SQUARE
1	1	10	100
2	4	11	121
3	9	12	144
4	16	13	169
5	25	14	196
6	36	15	225
7	49	20	400
8	64	25	625
9	81	30	900

For example, to find $\sqrt{81}$, note that 81 is the perfect square of 9, or $9^2 = 81$. Therefore, $\sqrt{81} = 9$.

Cube Root

Cube root is the procedural inverse of raising to a cube. If $2^3 = 8$, then $\sqrt[3]{8} = 2$. The cube root of $27 = 3$; $3^3 = 27$.

Algebra

Algebra is the branch of mathematics that focuses on addition, subtraction, multiplication, and division operations applied to variables, or unknowns, instead of specific numbers.

Here are these operations in algebraic form:

Operation	Algebraic Form
Addition: The sum of two numbers	$x + y$
Subtraction: The difference between two numbers	$x - y$
Multiplication: The product of two numbers	$x \times y$ *or* xy
Division: The quotient of two numbers	$\frac{x}{y}$

Algebraic Equations

An *equation* states that two quantities are equal. The solution to an equation is a number that can be substituted for the letter, or *variable*, to give a true statement.

For example, in the equation $x + 7 = 10$, if 5 is substituted for x, the equation becomes $5 + 7 = 10$, which is false. If 3 is substituted for x, the equation becomes $3 + 7 = 10$, which is true. Therefore, $x = 3$ is a solution for the equation $x + 7 = 10$.

An equation has been solved when it is transformed or rearranged so that a variable or unknown is on one side of the equal sign and a number is on the other side.

There are two basic principles that are used to transform equations:

1. The same quantity may be added to, or subtracted from, both sides of an equation.

   To solve the equation $x - 3 = 2$, add 3 to both sides:

   $$\begin{array}{rcl} x - 3 &=& 2 \\ \underline{+3} & & \underline{+3} \\ x &=& 5 \end{array}$$

   Adding 3 isolates x on one side and leaves a number on the other side. The solution to the equation is $x = 5$.

To solve the equation $y + 4 = 10$, subtract 4 from both sides (adding -4 to both sides will have the same effect):

$$\begin{array}{rcl} y+4 & = & 10 \\ \underline{-4} & & \underline{-4} \\ y & = & 6 \end{array}$$

The variable has been isolated on one side of the equation. The solution is $y = 6$.

2. Both sides of an equation may be multiplied by, or divided by, the same quantity.

To solve $2a = 12$, divide both sides by 2:

$$\frac{2a}{2} = \frac{12}{2}$$
$$a = 6$$

To solve $\frac{b}{5} = 10$, multiply both sides by 5:

$$5 \cdot \frac{b}{5} = 10 \cdot 5$$
$$b = 50$$

To solve equations containing more than one operation:

First, eliminate any number that is being added to or subtracted from the variable. Then eliminate any number that is multiplying or dividing the variable (A number that is multiplying the variable is called a *coefficient.*)

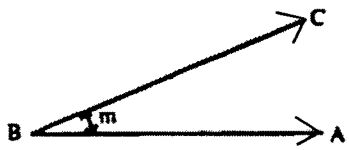
Solve:

$3x - 6$	$=$	9	
$\underline{+6}$		$\underline{+6}$	Adding 6 eliminates -6.
$3x$	$=$	15	
$\frac{3x}{3}$	$=$	$\frac{15}{3}$	Dividing by 3 eliminates the 3 that is multiplied by the x.
x	$=$	5	The solution to the original equation is $x = 5$.

Geometry

Angles

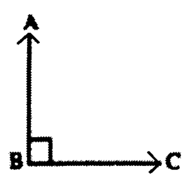
An *angle* is the figure formed by two rays meeting at a point.



The point B is the *vertex* of the angle, and $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are the *sides* of the angle. The symbol for an angle is ∠. The letter *m* stands for the measure of the angle in degrees.

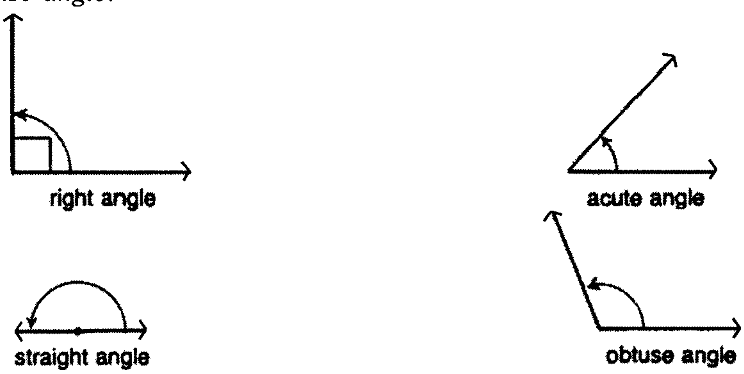
Types of Angles

1. When two straight lines intersect (cut each other), four angles are formed. If these four angles are equal, each angle is a *right angle* and contains 90°. The symbol is used to indicate a right angle, as shown below.

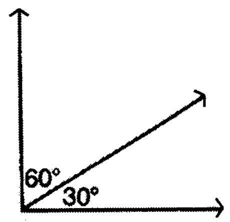


∠ABC is a right angle.

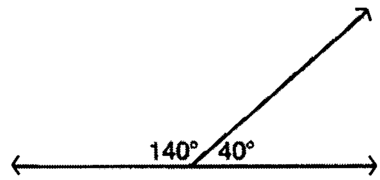
2. An angle that is smaller than a right angle is an *acute angle.*
3. If the two sides of an angle extend in opposite directions forming a straight line, the angle is a *straight angle* and measures 180°.
4. An angle that is bigger than a right angle (90°) and smaller than a straight angle (180°) is an *obtuse angle.*



Complementary angles are two angles whose measures sum to 90°. Each angle is the complement of the other. If an angle measures 30°, its complement measures 60°. If an angle measures $x°$, its complement measures $(90 - x)°$.



Complementary Angles

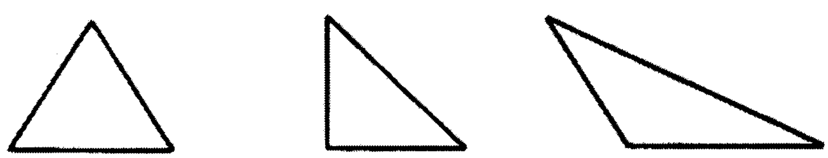


Supplementary Angles

Supplementary angles are two angles whose measures sum to 180°. Each angle is the supplement of the other. If an angle measures 140°, its supplement measures 40°. If an angle measures $x°$, its supplement measures $(180 - x)°$.

Triangles

A *triangle* is a closed, three-sided figure. The following figures are triangles.



The sum of the measures of three angles of a triangle is 180°.

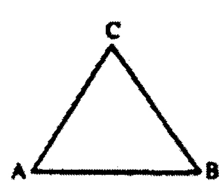
To find the measure of an angle of a triangle given the measure of the other two angles, add the measures and subtract their sum from 180°.

For example, if the measures of two angles of a triangle are 60° and 40°, the measure of the third angle is

$$180° - (60° + 40°) =$$
$$180° - 100° = 80°$$

A triangle with two congruent sides is called an *isosceles triangle.*

In an isosceles triangle, the angles opposite the congruent sides are also congruent.



If AC = BC, then $m\angle A = m\angle B$

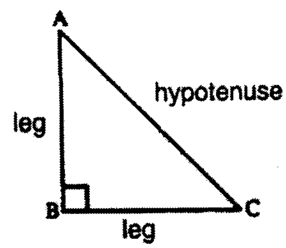
A triangle with all three sides congruent is called an *equilateral triangle.*

Each angle of an equilateral triangle measures 60°.

A triangle with a right angle is called a *right triangle.*

In a right triangle, the two acute angles are complementary.

In a right triangle, the side opposite *the right angle* is called the *hypotenuse* and is the longest side. The other two sides are called *legs.*



In right triangle ABC, $\overline{AC}$ is the hypotenuse. $\overline{AB}$ and $\overline{BC}$ are the legs.

The *Pythagorean theorem* states that in a right triangle, the square of the hypotenuse equals the sum of the squares of the legs.

In right triangle ABC: $(AC)^2 = (AB)^2 + (BC)^2$

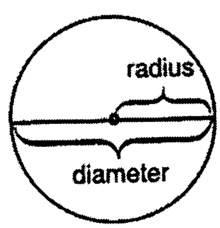
Circles

A *circle* is a closed plane curve, all points of which are equidistant from a point within called the center.

A complete circle contains 360°.

A *radius* of a circle is a line segment connecting the center with any point on the circle.

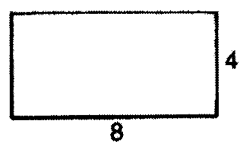
A *diameter* of a circle is a line segment connecting any two points on the circle and passing through the center of the circle. The diameter of any circle is twice the radius of that circle.



Perimeter

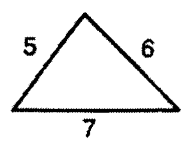
The *perimeter* of a two-dimensional figure is the distance around the figure. The perimeter of a rectangle equals twice the sum of the length and the width.

$$P = 2(l + w)$$
$$P = 2(8 + 4) = 2(12)$$
$$P = 24$$



The perimeter of a triangle is the sum of the three sides.

$$P = 7 + 6 + 5 = 18$$

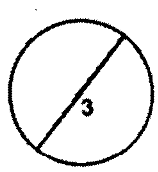


The perimeter of a circle is called the circumference. The circumference of a circle is equal to the product of the diameter multiplied by π.

The formula is $C = \pi d$

Pi (π) is a mathematical value equal to approximately 3.14 or $\frac{22}{7}$.

$$C = \pi d$$
$$C = \pi(3) = 3\pi$$



Area

In a two-dimensional figure, the total space within the figure is called the *area.*

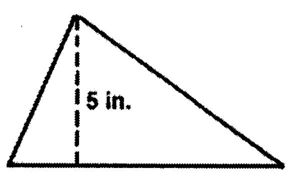
Area is expressed in square denominations, such as square inches, square centimeters, or square miles.

The area of a rectangle equals the product of the length (or base) multiplied by the width (or height).

$$A = lw$$
$$A = 9\,ft. \times 3\,ft.$$
$$A = 27\ sq.ft.$$

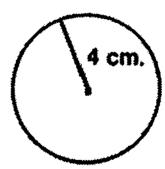
The area of a triangle is equal to one half the product of the base and the height. The height (or altitude) of a triangle is a line drawn from a vertical perpendicular to the opposite side, called the base.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(9 \text{ in.})(5 \text{ in.}) = \frac{45}{2}$$
$$A = 22\frac{1}{2} \text{sq. in.}$$



The area of a circle is equal to the radius squared multiplied by π.

$$A = \pi r^2$$
$$A = \pi(4 \text{ cm.})^2$$
$$A = 16\pi \text{ sq. cm.}$$



For some ASVAB questions, you may leave the area in terms of pi.

Sample Questions

Sample questions illustrating some of the types of questions found in the Mathematics Knowledge subtest follow. Explanatory answers are given at the end of this section and show how the correct answers are obtained.

1. The sum of $2\frac{5}{8}$, $3\frac{3}{16}$, $1\frac{1}{2}$, and $4\frac{1}{4}$ is

   1-A $9\frac{13}{16}$

   1-B $10\frac{7}{16}$

   1-C $11\frac{9}{16}$

   1-D $13\frac{3}{16}$

2. Which fraction is equal to 0.20?

   2-A $\frac{1}{5}$

   2-B $\frac{2}{7}$

   2-C $\frac{3}{16}$

   2-D $\frac{1}{50}$

3. Which of the following fractions is the least?

   3-A $\frac{3}{4}$

   3-B $\frac{5}{6}$

   3-C $\frac{7}{8}$

   3-D $\frac{19}{24}$

4. The product of $11\frac{2}{13}$ times $13\frac{7}{9}$ is most nearly

4-A 152.58
4-B 152.68
4-C 153.58
4-D 153.68

5. The sum of $\sqrt{81}$ and $\sqrt{25}$ is
5-A 106
5-B 86
5-C 24
5-D 14

6. Find the value of $\left(3\sqrt{2}\right)^2$

6-A $9\sqrt{2}$
6-B 18
6-C 24
6-D 36

7. $\sqrt[3]{216}$ is equal to
7-A 6
7-B 12
7-C 36
7-D 72

8. The fourth root of 81 is
8-A 324
8-B 27
8-C 9
8-D 3

9. The numerical value of 5! is
9-A 110
9-B 115
9-C 120
9-D 125

10. The numerical value of $\frac{4!}{3!}$ is
10-A .75
10-B 1.25
10-C 1.33
10-D 4

11. Which one of the following is a prime number?
    11-A 9
    11-B 11
    11-C 15
    11-D 21

12. The reciprocal of 4 is
    12-A .25
    12-B .40
    12-C 1.25
    12-D 1.40

13. 1,000 is equivalent to
    13-A 10^2
    13-B 10^3
    13-C 10^4
    13-D 10^5

14. $10^3 \times 10^4 =$
    14-A 10^7
    14-B 10^{12}
    14-C 100^7
    14-D 100^{12}

15. When +5 is added to –7, the sum is
    15-A +2
    15-B –2
    15-C +12
    15-D –12

16. Find the product of (–5)(–4)(–3).
    16-A +12
    16-B –12
    16-C +60
    16-D –60

17. Solve the following: $\frac{5}{9}(41+40)-40=$
    17-A 55
    17-B 5
    17-C 22.30
    17-D 73.80

18. If you subtract –1 from +1, the result will be
    - 18-A –2
    - 18-B –1
    - 18-C +1
    - 18-D +2

19. If $a + 6 = 7$, then a is equal to
    - 19-A 0
    - 19-B $\frac{7}{6}$
    - 19-C +1
    - 19-D –1

20. If $4y = 12$, then $y =$
    - 20-A $\frac{1}{4}$
    - 20-B $\frac{1}{3}$
    - 20-C 3
    - 20-D 8

21. If 50% of $x = 66$, then $x =$
    - 21-A 132
    - 21-B 99
    - 21-C 66
    - 21-D 33

22. $8 \times 8 = 4^x$. Find x.
    - 22-A 1
    - 22-B 2
    - 22-C 3
    - 22-D 4

23. If $2^{n-3} = 32$, then n equals
    - 23-A 5
    - 23-B 6
    - 23-C 7
    - 23-D 8

24. What percent of a is b?

24-A $\frac{b}{a}$

24-B $\frac{a}{b}$

24-C $\frac{100b}{a}$

24-D $\frac{100a}{b}$

25. Using the formula $A = P(1 + rt)$, find A when $P = 500$, $r = .03$, and $t = 15$.

25-A 625
25-B 725
25-C 795
25-D 800

26. If $a = 5b$, then $\frac{3}{5}a =$

26-A $\frac{5}{3}b$

26-B $\frac{3}{5}b$

26-C $3b$

26-D $\frac{b}{3}$

27. If you multiply $x + 3$ by $2x + 5$, what will be the coefficient of x?

27-A 11
27-B 10
27-C 9
27-D 6

28. $\frac{x-2}{x^2-6x+8}$ can be simplified to

28-A $\frac{1}{x-4}$

28-B $\frac{1}{x-2}$

28-C $\frac{1}{x+4}$

28-D $\frac{1}{x+2}$

29. If $2x = 3y$ and $5x + y = 34$, $y =$

29-A 4
29-B 5
29-C 6
29-D 7

30. Solve for x: $x + y = a$
$x - y = b$

30-A $a + b$
30-B $a - b$
30-C $\frac{1}{2}(a+b)$
30-D $\frac{1}{2}(a-b)$

31. Solve for x: $\frac{x+1}{8} = \frac{28}{32}$

31-A 5
31-B 6
31-C 7
31-D 8

32. If $\frac{a}{b} \times \frac{b}{c} \times \frac{c}{d} \times \frac{d}{e} \times x = 1$, then x must be equal to

32-A $\frac{a}{e}$

32-B $\frac{e}{a}$

32-C $\frac{1}{a}$

32-D $\frac{1}{e}$

33. If $\frac{a}{b} = \frac{3}{4}$, then $12a =$

33-A $3b$
33-B $6b$
33-C $9b$
33-D $12b$

34. The average of two numbers is A. If one of the numbers is x, the other number is

34-A $\frac{A}{2} - x$

34-B $\frac{A+x}{2}$

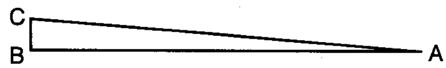
34-C $A - x$
34-D $2A - x$

35. Two angles that are both congruent and supplementary are
35-A acute angles.
35-B obtuse angles.
35-C right angles.
35-D straight angles.

36. In one hour, the minute hand of a clock rotates through an angle of
36-A 45°
36-B 90°
36-C 180°
36-D 360°

37. At 6:00 a.m., the angle between the hands of the clock is
37-A 90°
37-B 120°
37-C 180°
37-D 360°

38. In the triangle given below, if $m\angle B = 90°$,



38-A $\overline{AB}$ is longer than $\overline{AC}$.
38-B $m\angle ABC$ is less than $m\angle ACB$.
38-C $m\angle ABC = m\angle ACB$.
38-D $m\angle ABC$ is greater than $m\angle ACB$.

39. A circle is inscribed in a square whose side is 6. What is the circumference of the circle in terms of π?
39-A 3π
39-B 6π
39-C 9π
39-D 12π

40. The hypotenuse of a right triangle whose legs are 5″ and 12″ is
40-A 7″
40-B 13″
40-C 14″
40-D 17″

41. Approximately how many meters will a point on the rim of a wheel travel if the wheel makes 50 rotations and its radius is 1 meter?
41-A 314
41-B 298
41-C 283
41-D 157

42. The area of a square is 36 square inches. If the side of this square is doubled, the area of the new square will be
42-A 72 square inches.
42-B 144 square inches.
42-C 216 square inches.
42-D 244 square inches.

43. The circumference of a circle that has a radius of 70 feet is most nearly
43-A 440 feet.
43-B 660 feet.
43-C 690 feet.
43-D 15,300 feet.

44. The distance between two points on a graph whose rectangular coordinates are (2, 4) and (5, 8) is most nearly
44-A 5.0
44-B 5.5
44-C 6.0
44-D 6.5

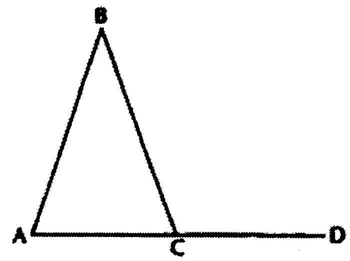
45. In triangle ABC, AB = BC and $\overline{AC}$ is extended to D. If angle BCD measures 110°, find the number of degrees in angle B.

45-A 20°

45-B 40°

45-C 60°

45-D 80°



Answers and Explanations

1-C Arrange the numbers in proper vertical columns, then rename fractions as equivalent fractions having the same common denominator and, finally, add.

The sum of $10\frac{25}{16}$ is actually $11\frac{9}{16}$.

2-A $.2 = \frac{2}{10} = \frac{1}{5}$

3-A Rename all fractions as 24ths and then compare numerators.

$A = \frac{18}{24},\ B = \frac{20}{24},\ C = \frac{21}{24},\ D = \frac{19}{24}$

4-D Rename the mixed numbers as improper fractions and then multiply: $\frac{145}{13} \times \frac{124}{9} = \frac{17980}{117} = 153.68$

5-D $\sqrt{81} = 9;\ \sqrt{25} = 5;\ 9 + 5 = 14$

6-B $\left(3\sqrt{2}\right)^2 = 3\sqrt{2} \times 3\sqrt{2} = 9 \times 2 = 18$

7-A $6 \times 6 \times 6 = 216$

8-D $3 \times 3 \times 3 \times 3 = 81$

9-C 5 factorial is $5 \times 4 \times 3 \times 2 \times 1 = 120$

10-D The factorial of a natural number is the product of that number and all the natural numbers less than it. $4! = 4 \times 3 \times 2 \times 1 = 24$.

$3! = 3 \times 2 \times 1 = 6$

$\frac{4!}{3!} = \frac{24}{6} = 4$

11-B Of the numbers given, only 11 has no other factor except 1 and itself. $3 \times 3 = 9; 3 \times 5 = 15; 7 \times 3 = 21$.

12-A If the product of two numbers is 1, either number is called the reciprocal or multiplicative inverse of the other. For example, since $4 \times \frac{1}{4} = 1$, 4 is the reciprocal of $\frac{1}{4}$ and $\frac{1}{4}$ is the reciprocal of 4. $\frac{1}{4}$ is equivalent to .25.

13-B $10 \times 10 \times 10 = 1{,}000$

14-A $10^3 = 1{,}000$; $10^4 = 10{,}000$; $1{,}000 \times 10{,}000 = 10{,}000{,}000$ or 10^7. To multiply numbers of the same base, add the exponents. $10^3 \times 10^4 = 10^{(3+4)} = 10^7$.

15-B To add numbers with different signs, subtract the magnitude of the numbers and use the sign of the number with the greater magnitude.

16-D If there is an odd number of negative factors when multiplying, the product is negative. $(-5)(-4)(-3) = -60$.

17-B $\left(\frac{5}{9} \times 81\right) - 40 = 45 - 40 = 5$

18-D Subtracting −1 from +1, change −1 to +1 and add to +1 = +2.

19-C $a = 7 - 6 = +1$

20-C $y = \frac{12}{4} = 3$

21-A $\frac{1}{2}$ of $x = 66$; $x = 66 \times 2 = 132$

22-C $8 \times 8 = 64$; $4 \times 4 \times 4 = 64$; $x = 3$

23-D $2^5 = 32$; $n - 3 = 5$; $n = 8$

24-C $\frac{b}{a} \times 100 = \frac{100b}{a}$

25-B $A = 500(1 + .03 \times 15) = 500(1 + .45) = 500(1.45) = 725$

26-C $\frac{3}{5} \times 5b = 3b$

27-A

$$\begin{array}{r} x+3 \\ \underline{2x+5} \\ 2x^2+6x \\ \underline{+5x+15} \\ 2x^2+11x+15 \end{array}$$

28-A The factors of $x^2 - 6x + 8$ are $(x - 4)$ and $(x - 2)$.

Therefore, $\frac{x-2}{x^2-6x+8} = \frac{x-2}{(x-4)(x-2)} = \frac{1}{(x-4)}$.

29-A Solve for x: $2x = 3y$

$$x = \frac{3y}{2}$$

Substitute in the second equation and solve for y:

$$5\left(\frac{3y}{2}\right) + y = 34$$
$$\frac{15y}{2} + y = 34$$
$$15y + 2y = 68$$
$$17y = 68$$
$$y = 4$$

30-C Add the two equations to eliminate y:

$$x + y = a$$
$$x - y = b$$
$$2x = a + b$$

Solve for x:

$$x = \frac{a+b}{2}$$

31-B Solve for x:

$$\frac{x+1}{8} = \frac{28}{32}$$

$$(x+1)32 = 8 \times 28$$
$$32x + 32 = 224$$
$$32x = 224$$
$$32x = 192$$
$$x = \frac{192}{32} = 6$$

32-B Divide common factors, and then solve.

$$\frac{a}{\not b}\times\frac{\not b}{\not c}\times\frac{\not c}{\not d}\times\frac{\not d}{e}\times x=1$$

$$\frac{a}{e}\times x=1$$

$$\frac{ax}{e}=1$$

$$ax=e$$

$$x=\frac{e}{a}$$

33-C Solve for a: $\frac{a}{b}=\frac{3}{4}$

$$4a=3b$$

$$a=\frac{3b}{4}$$

$$12a=\overset{3}{\not 12}\left(\frac{3b}{\not 4}\right)$$

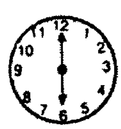
$$12a=9b$$

34-D Let x = one of the numbers and y = the other number. $\frac{x+y}{2}=A; x+y=2A; y=2A-x$.

35-C If the angles are congruent and supplementary, they must be of the same measure and add up to 180°. Each measures 90° or a right angle.

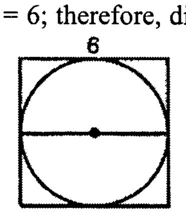
36-D In 1 hour, the minute hand rotates a full circle of 360°.

37-C At 6 a.m., one hand is at 6 and the other is at 12, forming a straight angle or 180°.



38-D In this right triangle, angle ABC is a right angle. Each of the measures of the other angles in the triangle must be less than 90°. $\overline{AC}$, the hypotenuse, is longer than either leg of the triangle.

39-B Side = 6; therefore, diameter = 6. Circumference = π × diameter = 6π.



40-B The Pythagorean theorem states that for any right triangle, the sum of the square of the legs is equal to the square of the length of the hypotenuse. $5^2 + 12^2 = h^2$, $25 + 144 = h^2$, $h^2 = 169$. $\sqrt{169} = 13$. $h = 13$. The correct answer is 13″.

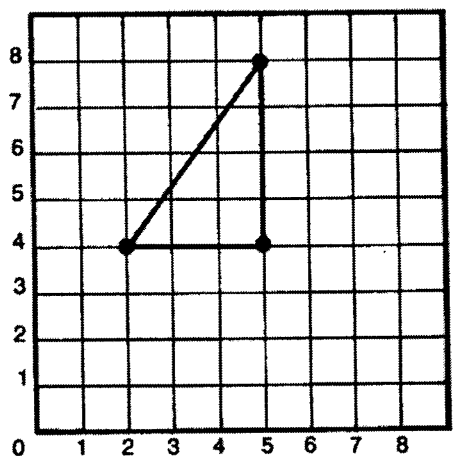
41-A If the radius of the wheel is 1 meter, its diameter is 2 meters. The circumference is $\pi \times$ diameter $= 2 \times 3.14$. The distance traveled is $50 \times 2 \times 3.14 = 100 \times 3.14 = 314$.

42-B If the area of a square = 36 square inches, the side of the square = 6 inches. If doubled to 12 inches, the area of the new square will be 12 inches by 12 inches = 144 square inches.

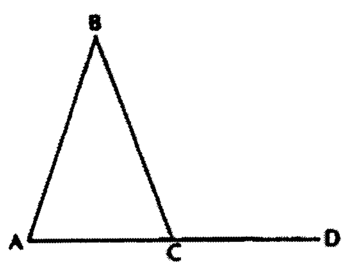
43-A If the radius is 70 feet, the diameter is 140 feet. Circumference = $\pi \times$ diameter =

$$140 \times \frac{22}{7} = 440 \text{ feet.}$$

44-A As shown in the following graph, we have a right triangle with one leg of 3 and the other leg of 4. Using the Pythagorean theorem, the hypotenuse, or the distance between the two points, is obtained as follows: $h^2 = 3^2 + 4^2$; $h^2 = 9 + 16$; $h^2 = 25$; $h = \sqrt{25}$; $h = 5$.



45-B



ACD is a straight line. Therefore, if m∠ BCD = 110°, m∠ BCA = 70°. AB = BC. Therefore, triangle ABC is an isosceles triangle and m∠ BAC = m∠ BCA = 70°. The sum of the measures of the angles of a triangle = 180°. Therefore, m∠ ABC = 180° – (70° + 70°) = 180° – 140° = 40°.