Chapter 7

Mathematics Knowledge

In This Chapter

- Dealing with slightly harder math problems
- Laying down the terms
- Practicing your technique

The Mathematics Knowledge subtest evaluates your understanding of sophisticated mathematical principles and your ability to make correct calculations using those principles.

To qualify for certain jobs in the military, you have to score well on the Mathematics Knowledge subtest. You also have to do well on this subtest (which is part of the Armed Forces Qualifying Test) in order to enlist. Turn to Appendix A to find out more about the subtest scores needed for specific military jobs.

Bringing Up the Background Info

The Mathematics Knowledge subtest consists of 25 questions covering a wide range of mathematical concepts. You have 24 minutes to complete the subtest. You don't necessarily have to rush through each calculation, but the pace you need to set (about a minute per question) doesn't exactly give you time to daydream about what you're having for dinner. You have to focus and concentrate to solve each problem quickly and accurately.

This subtest contains questions expressed in mathematical terms and questions consisting of word problems. But, usually, the Mathematics Knowledge subtest has far fewer word problems than the Arithmetic Reasoning subtest features. (For the straight scoop on the Arithmetic Reasoning subtest, check out Chapter 6.) See, the test-makers do give you a break once in a while.

Determining the general content

Most of the time, the Mathematics Knowledge subtest only contains one or two questions testing each specific mathematical concept. For example, one question may ask you to multiply fractions, the next question may ask you to solve an inequality (a mathematical inequality, not a political or social inequality), and the question after that may ask you to find the value of an exponent. (If we've freaked you out with the last sentence, calm down. We cover inequalities in the "Solving inequalities" section and exponents in the "Minding Your P's and Q's and X's and Y's: Algebra Review" section later in this chapter. Flip back to Chapter 6 for basic information about fractions.)

All this variety forces you to constantly shift your mental gears to quickly deal with different concepts. You can look at this situation from two perspectives. These mental gymnastics can be difficult and frustrating, especially if you know everything about solving for x but nothing about deriving a square root. But variety can also be the spice of life, as your grandma may say. If you don't know how to solve a specific type of problem, this oversight may only cause you to get one question wrong (or maybe two, but think positive). If you use the guessing techniques we describe in the "Guessing your way to better odds" section later in this chapter, your odds of getting the question right are higher, even if you don't know anything about the concept.

Moving uptown with slightly more sophisticated sums

The Mathematics Knowledge subtest requires more extensive mathematical knowledge than the Arithmetic Reasoning subtest. Sorry about that. On the Mathematics Knowledge subtest, the questions relate more to algebra and geometry than to basic mathematical operations like adding and multiplying. Although you do have to add and multiply on this subtest too.

Ordering your operations: Parentheses take precedence

Many of you could become military mathematicians if the ASVAB only contained problems like $3 \times 5 = ?$ and $15 + 19 = ?$. But, instead, it features problems that can be a bit more confusing such as $5 + (16 \times 2) = ?$.

When you see parentheses in a math problem, the calculation in the parentheses should be done first. For example, in $5 + (16 \times 2) = ?$, you first multiply 16 by 2 to arrive at 32, and then you add the 5 to come up with a total of 37. You get a different (and wrong) answer if you simply calculate from left to right: $5 + 16 = 21$. $21 \times 2 = 42$.

REMEMBER

To figure out which mathematical operation you should perform first, second, third, and so on, follow these rules, otherwise known as the *order of operations:*

1. **Grouping symbols take precedence.** Do the operation indicated by grouping symbols first. The fraction bar (— or /) is a grouping symbol. So if you have the problem $\frac{1+2}{3} = ?$, you add the numbers above or to the left of the fraction bar and then divide. The answer is $\frac{3}{3} = 1$.

   The square root sign ($\sqrt{\ }$) is also a grouping symbol, so you would solve for the square root before doing any other operation in the problem. (For more on fractions, report for duty at the Chapter 6. And to get your fill of square roots, march on over to "Getting to the root, the whole root, and nothing but the root" section later in this chapter.)

2. **Parentheses come next.** Do all the work inside of parentheses after you've finished with the grouping symbols.

3. **Multiplication and division are next.** You always do these operations in left-to-right order (just like you read).

4. **Addition and subtraction are last.** Perform these operations from left to right as well.

EXAMPLE

Ordering in action

The following example shows you how to perform a problem's operations in order:

$$(15 \div 5) \times 3 + (18 - 7) = ?$$

Do the work in parenthesis first (because no grouping symbols are used in this problem). The result is

$$3 \times 3 + 11 = ?$$

Then do division and multiplication (in this problem, only multiplication is needed). You end with

$$9 + 11 = ?$$

Finally, do the addition and subtraction (in this problem, only addition is needed). Your final answer is 20.

Translating Terminology Tips

In order to understand what each problem on the Mathematical Knowledge subtest asks you to do, you must understand certain mathematical terms, such as the ones we cover in the following sections.

Displaying mathematical reciprocity

A *reciprocal* is the number by which another number can be multiplied to produce 1. For example, the reciprocal of 3 is $\frac{1}{3}$. If you multiply 3 times $\frac{1}{3}$ you produce 1. The reciprocal of $\frac{1}{6}$ is $\frac{6}{1}$ (which is the same thing as 6). $\frac{1}{6} \times 6 = 1$. Get the idea?

Hanging out at mathematical bases, not army bases

A *base* is a number that is used as a factor at least two times. For instance, the term 4^3 (which could be written $4 \times 4 \times 4$, and in which 4 is a factor three times) has a base of 4.

Working at the factorial factory

A *factorial* is the product of a whole number and all the whole numbers less than it. So 6 factorial is $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$. A factorial is represented by an exclamation point! $6! = 720$.

A factorial helps you determine *permutations* — all the different possible ways an event might turn out. For example, if you want to know how many different ways six runners could finish a race (permutation), you would solve for 6.

Rounding the bend

Rounding a number means limiting a number to a few (or no) decimal places. For example, if you have a $1.97 in change in your pocket, you may say, "I have about two dollars." The rounding process simplifies mathematical operations.

Often, numbers are rounded to the nearest tenth. The ASVAB may ask you to do this. For any number 5 and over, round up; for any number under 5, round down. Thus, 1.55 can be rounded up to 1.6, and 1.34 can be rounded down to 1.3.

Discovering roots you never knew you had

The *square root* of a number is the number, which, when multiplied by itself *(squared),* equals the original number. For example, the square root of 36 is 6. If you square 6, or multiply it by itself, you produce 36. (We provide more on square roots in the "Getting to the root, the whole root, and nothing but the root" section later in this chapter.)

Minding Your P's and Q's and X's and Y's: Algebra Review

Remember sitting in high-school algebra class and saying, "I'll never use this in real life"? Well, the ASVAB disagrees. (And, if you never took algebra in school, the ASVAB doesn't care.) It extensively tests your algebra skills. But stick with us and you'll be fine.

Algebra is a way to put problems into mathematical language using the simplest mathematical terms possible.

In algebra, you often hear about "solving for x" or "solving for the unknown," but what's the unknown? That's an easy one. The *unknown* is simply the answer you want find. Check out this example:

If you want to go to the stockcar races in your hometown, and one ticket costs $35, how much will it cost to buy tickets for a family of 4?

You can express this problem in terms of x, with x being how much it will cost to buy tickets for the whole family: x equals 4 (the number of people in the family) times $35 (the ticket price). Written a bit more formally, the equation looks like this: $x = 4 \times 35$ *or* $4 \times 35 = x$.

What if you don't know how much the stockcar tickets cost? You can express this missing piece of information in an equation as well: x (how much it will cost to buy tickets for the whole family) equals 4 (the number of people in the family) times p (the price of one ticket to the race). Once again, written a bit more formally, the equation looks like this: $x = 4 \times p$.

You can remove the multiplication symbol in algebraic expressions when using a combination of letters and numbers. Therefore, the equation $x = 4 \times p$ can also be written $x = 4p$. The multiplication symbol is implied.

The letters in an algebra problem are commonly called *variables,* meaning that the number they stand for *varies,* or changes.

Talking like a math professor: Terminology you need to know

Special algebra terms are used to describe how numbers function and how they relate to each other. Knowing what these terms mean is important to your ASVAB success.

- **Prime number:** A whole number that can be divided evenly by itself and by 1 but not by any other number, which means that it has exactly two *factors.* (Check out the definition of *factor* a bit later in this list.) Examples of prime numbers are 2, 5, and 11.
- **Composite number:** A whole number that can be divided evenly by itself and by 1, as well as by one or more other whole numbers, which means that it has more than two factors. Examples of composite numbers are 6, 8, and 9.
- **Factors:** Numbers that can divide into a composite number. To *factor* a composite number, you simply determine the numbers that you can divide into it. For example, 8 can be divided by the numbers 2 and 4 (in addition to 1 and 8), so 2 and 4 are factors of 8.
- **Exponents:** You can think of exponents as a shorthand method of indicating multiplication. For example, 15×15 can also be expressed as 15^2, which is also known as "15 squared" or "15 to the second power." The small number (^{2}) written slightly above and to the right of a number is called the *exponent.* An exponent indicates the number of times you multiply the number it accompanies by itself — 15^2 (15×15) is *not* the same as 15×2.

  To express $15 \times 15 \times 15$ using this shorthand method, simply write it as 15^3, which is also called "15 cubed" or "15 to the third power." Again, 15^3 isn't the same as 15×3.

Solving for x: The algebra equation

Algebra problems are equations, which means that the quantities on both sides of the equal sign are equal — they're the same. We can say that $2 = 2$. We can say that $1 + 1 = 2$. And we can say that $3 - 1 = 2$. In all these cases, the quantities are the same on both sides of the equal sign. So, if $x = 2$, then x *is* 2 because the equal sign says so. And, like your mother, you should always listen to the equal sign.

Solving one-step equations involving addition and subtraction

If $x + 1 = 2$, then x must be 1, because only 1 added to 1 results in 2. So far, so simple, so good. But what if the equation is a little more complicated:

$$x + 47{,}432 = 50{,}000$$

To find out what x equals, which solves the problem, you need to isolate x on one side of the equal sign. To get that job done, you have to move any other numbers on the x side of the equal sign to the other side of the equal sign.

By looking at the x side of the equation, you can see that it's an addition problem. To move the number on the x side to the opposite side, you have to perform the inverse operation. The inverse operation of addition is subtraction. (For a full rundown on inverse operations, check out Chapter 6.) So, to move the 47,432 from the x side to the non-x side of the equation, simply subtract it from both sides:

$$x + 47{,}432 - 47{,}432 = 50{,}000 - 47{,}432$$

Performing these operations removes the 47,432 from the x side of the equation ($47{,}432 - 47{,}432 = 0$, so that side of the equation is $x + 0$ or simply x) and gives you 2,568 on the non-x side of the equation ($50{,}000 - 47{,}432 = 2{,}568$). You're left with the final answer:

$$x = 2{,}568$$

To double-check that this answer is correct, plug your answer into the original problem:

$x + 47{,}432 = 50{,}000$

$2{,}568 + 47{,}432 = 50{,}000$

If you plug the answer in and it doesn't work, you've made an error in your calculations. Start again; remember that you're trying to isolate x on one side of the equation.

You can perform any calculation on either side of an equation as long as you do it to both sides of the equation. That keeps the equation *equal.*

Multiplying and dividing using integers

An *integer* is any positive or negative whole number or zero. The ASVAB often requires you to work with integers, as in $-6x = 36$. (Don't forget, $6x$ is the same thing as $6 \times x$.) In multiplication and division, if the signs of the two terms being operated on are both plus (positive numbers) or both minus (negative numbers), the answer is a positive number. If one number is negative and the other is positive, the answer is negative.

To solve this problem, $-6x = 36$, you need to isolate x, so perform an inverse operation (remember, the inverse operation of multiplication is division):

$-6x \div -6 = 36 \div -6$

$x = -6$

The answer is a negative number because the two terms, 36 and –6, have different signs.

In an algebra equation, if the same letter is used more than once, it stands for the same number. Thus, in $3x + 2x = 10$, the first x will never ever be a different number from the second x. In this case, $x = 2$ (both times).

You can only combine like terms when operating on algebraic expressions: $3x + 3x = 6x$, but $3x + 3y$ does *not* equal $6xy$, nor does $x^2 + x^3 = x^5$.

Watching the x files: Multistep equations

Not all algebra problems have one-step solutions. (That would be too easy, and you wouldn't sweat nearly as much. The ASVAB test-makers can't have that, can they?) Solving algebra problems on the ASVAB often requires you to perform several steps.

An example of a multistep equation is when x shows up on both sides of the equal sign. Then you have to get rid of x from one side of the equation by moving an x from one side to the other. You do this by performing the inverse operation.

Suppose you want to solve this equation:

$3x + 3 = 9 + x$

To remove the x from one side of the equation, perform the inverse operation:

$3x + 3 - x = 9 + x - x$

This equation can also be stated as

$3x + 3 - 1x = 9 + 0$

Doing the subtraction results in

$2x + 3 = 9$

To finish solving the problem, subtract 3 from each side of the equation:

$$2x + 3 - 3 = 9 - 3$$

$$2x = 6$$

Then divide both sides by 2:

$$2x \div 2 = 6 \div 2$$

$$x = 3$$

REMEMBER: When you have a variable by itself, such as *x*, it's always equal to 1 times that variable (or one of that variable), like 1*x*, even if the 1 isn't written out. In fact, any number is equal to 1 times itself, so you could also say $2 = 2 \times 1$. Sometimes this comes in handy when you're solving those algebra problems.

Getting to the root, the whole root, and nothing but the root

A *square root* is the factor (see the "Talking like a math professor: Terminology you need to know" section earlier in this chapter) of a number that, when multiplied by itself, produces the number. Take the number 36, for example. One of the factors of 36 is 6. If you multiply 6 by itself (6×6), you come up with 36, so 6 is the square root of 36. 36 has other factors such as 18. But, if you multiply 18 by itself (18×18), you get 324, not 36. So 18 is *not* the square root of 36. Any number can only have one square root.

Rooting out the details

Only a few numbers, called *perfect squares,* have exact square roots. All the rest have square roots that include decimals that go on forever and have no pattern that repeats (nonrepeating, nonterminating decimals), so they're called *irrational numbers.*

The sign for a square root is called the *radical sign.* It looks like this: $\sqrt{\ }$. Here's how you use it: $\sqrt{36}$ means "the square root of 36" — in other words, 6. We know, we know — simply saying "6" is easier, but we're not mathematical geniuses.

Finding square roots

To find the square root of a number, make an educated guess and then verify your results. If you have to find the square root of a number that isn't a perfect square, the ASVAB usually asks you to find the square root to the nearest tenth.

To use the educated-guess method, you have to know the square roots of a few perfect squares. For example, you should know that the square root of 25 is 5 ($5 \times 5 = 25$) and that the square root of 49 is 7 ($7 \times 7 = 49$). One good way to do this is to learn the squares of the square roots 1 through 12. 1 is the square root of 1; 2 is the square root of 4, 3 is the square root of 9, and so on.

Suppose you run across this problem on the ASVAB: $\sqrt{54}$. You know that the square root of 49 is 7, and 54 is slightly greater than 49. You also know that the square root of 64 is 8, and 54 is slightly less than 64. So, if the number 54 is somewhere between 49 and 64, the square root of 54 is somewhere between 7 and 8.

Because 54 is closer to 49 than to 64, the square root will be closer to 7 than to 8, so you can try 7.3 as the square root of 54. Multiply 7.3 by itself. $7.3 \times 7.3 = 53.29$, which is very close to 54. Then try multiplying 7.4 by itself to see if it's any closer to 54. $7.4 \times 7.4 = 54.76$, which isn't as close to 54 as 53.29. So 7.3 is the square root of 54 to the nearest tenth.

However, the wonderful world of math is also home to concepts like *cube roots, fourth roots,* and so on. These roots are a factor of a number, which, when *cubed* (multiplied by itself three times), taken to the *fourth power* (multiplied by itself four times), and so on, produce the original number. A few examples seem to be in order: The cube root of 27 is 3. If you cube 3 (also known as raising it to the *third power* or multiplying $3 \times 3 \times 3$), the product is 27. The fourth root of 16 is that number which, when multiplied by itself four times, equals 16. Any guesses? Drumroll, please: 2 is the fourth root of 16 because $2 \times 2 \times 2 \times 2 = 16$. You may be asked to multiply exponents to the fifth power, the sixth power, and so on.

Covering All the Angles: Geometry Review

Geometry is the branch of mathematics that makes grown adults cry — end of discussion. What? You want a more specific explanation of geometry than that? Okay, here goes. Geometry is the branch of mathematics concerned with measuring things and defining the properties of and relationships between and among shapes, lines, points, angles, and other such objects. Hey, don't blame us; you asked for it.

Measuring by degrees and minutes

Arcs, circles, triangles, and angles are measured in degrees and (not very often) in minutes (which are smaller than degrees). A circle has 360 degrees (360°); so does a *quadrilateral* (shapes with four sides like a square or rectangle). Therefore, any arc or angle that isn't a complete circle or quadrilateral measures less than 360°.

Outlining angles

Angles are formed when two lines intersect at a point. Angles are measured in degrees. The greater the number of degrees, the wider the angle is. Thus, a *straight line* is 180°. A *right angle* is exactly 90°. (The symbol for a right angle on a drawing is a "half-box" with a vertical line and a horizontal line connecting; see Figure 7-1 for an example.) An *acute angle* is more than 0° and less than 90°. An *obtuse angle* is more than 90° but less than 180°. *Complementary angles* are two angles that equal 90° when added together. *Supplementary angles* are two angles that equal 180° when added together. (You can take a look at the different types of angles in Figure 7-1.)

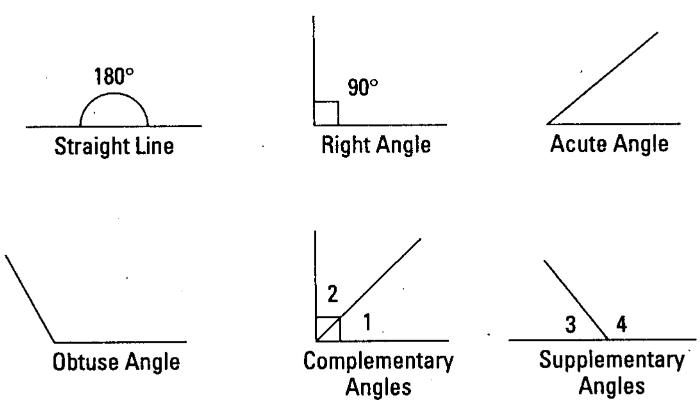

Figure 7-1: Different types of angles.

Pointing out triangle types

A *triangle* consists of three straight lines whose three angles always add up to 180°. The sides of a triangle are called *legs*. Triangles can be classified according to the relationship between their angles or the relationship between their sides, or some combination of these relationships. Check out Figure 7-2 to see what these triangles look like.

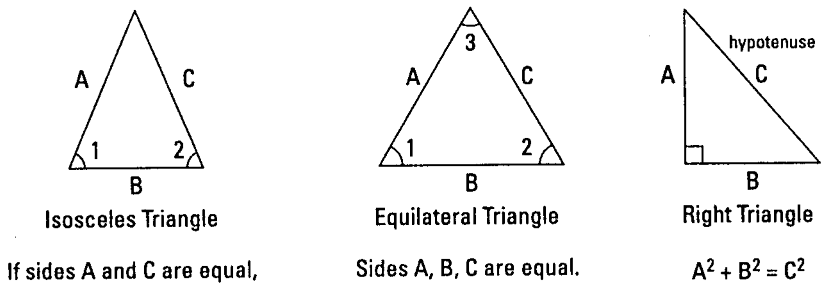

Figure 7-2: Different types of triangles.

- **Isosceles:** Has two equal sides. The angles opposite the equal sides are also equal.
- **Equilateral:** Has three equal sides. All the angles measure 60°.
- **Right:** Has one right angle (90°). Therefore, the remaining two angles are complementary (add up to 90°). The side opposite the right angle is called the *hypotenuse*, which is the longest side of a right triangle.

You can find the *perimeter* — the distance around a shape — of a triangle by adding together the length of the three sides.

The *area* — the space within a shape — of a triangle is one-half the product of the base (the bottom or the length) and the height (the tallest point of the triangle), or $\frac{1}{2}bh$.

Walking around the quad

Quadrilaterals — shapes with four sides — all contain angles totaling 360°. Many different types of quadrilaterals exist: *Squares* have four sides of equal length, and all the angles are right angles; *rectangles* have all right angles; *rhombuses* have four sides of equal length, but the angles don't have to be right angles; *trapezoids* have at least two sides that are parallel; and *parallelograms* have opposite sides that are parallel, and their opposite sides and angles are equal. (Figure 7-3 illustrates these quadrilaterals.)

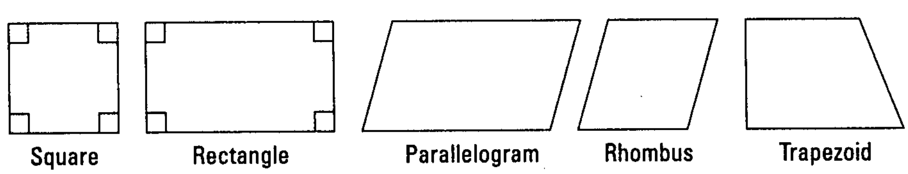

Figure 7-3: Different types of quadrilaterals.

To determine the *perimeter* of a quadrilateral, simply add the length of all the sides.

The *area* of a rectangle (including squares) can be determined by multiplying the length times the width.

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Mapping out the area of a circle

Determining the area of a circle also requires the use of π.

Area = $\pi \times$ the square of the circle's radius

or

$A = \pi r^2$

Thus, to determine the area of a 9-inch-diameter pie, multiply π by the square of 4.5, Why 4.5 and not 9? Remember, the radius is always half the diameter, and the diameter is 9 inches.

$A = \pi r^2$

$A \approx 3.14 \times 4.5^2$

$A \approx 3.14 \times 4.5 \times 4.5$

$A \approx 3.14 \times 20.25$

$A \approx 63.585$ inches

Filling 'er up: Calculating Volume

Volume is the space a solid (three-dimensional) shape takes up. You can think of volume as how much a shape would hold if you poured water into it. Volume is measured in cubic units.

The formula for finding volume depends on the object. For rectangular objects, you multiply length times width (depth) times height. This is possible because the length, width, and height of a rectangle are consistent throughout the whole shape. Thus, $V = lwh$.

For a box that measures 5-feet long, 6-feet deep and 2-feet tall, you simply multiply $5 \times 6 \times 2$ to arrive at a volume of 60 cubic feet.

For a cylinder, which has two circles for its bases, the calculation is $V = \pi r^2 h$ or, in good-old-fashioned plain English, volume equals pi times the radius squared times height.

For a cylinder that has a radius of 2 inches and a height of 10 inches, here's the deal: Multiply the value of pi (3.14) times 4 (which is the radius squared) times 10, or $3.14 \times 4 \times 10 = 125.6$ cubic inches.

Finding Calculation Clues like Starsky and Hutch

In this section, we bring you up to speed on how to solve problems that the Mathematics Knowledge subtest commonly throws at its victims.

Factoring factors into the equation

Now and then, the ASVAB gives you a *product* (the answer to a multiplication problem), and you have to find the original numbers that were multiplied together to produce that product. This process is called *factoring*. Take, for example, this product:

$4xy + 2x^2$

To factor this number, you first find the *highest common factor* — the highest number that evenly divides all the terms in the expression. In this case, the highest number that divides into both terms is 2.

But wait. You have to figure out the common factors for the variables too. In this case, the highest variable that divides into both xy and x^2 is x.

Okay. Take what you know to this point, and you can see that the highest common factor is $2x$. So far, so good. Now divide $2x$ into both terms in the expression; doing so produces the factors of $2x(2y + x)$.

You use factors when you combine like terms and add fractions.

Making alphabet soup: When x = 1, yru confused?

Algebra questions often ask you to solve for x or solve for an unknown. These questions can be expressed as, for example, $x = 2 + 3$. You simply isolate the unknown on one side of the equation and solve the other side to learn what x equals. In this case, x equals 5. We cover the topic of solving for unknowns in more depth in the section, "Minding Your P's and Q's and X's and Y's: Algebra Review," earlier in this chapter.

But now we come to the dreaded quadratic equation, which the Mathematical Knowledge subtest may ask you to solve. What's a *quadratic equation?* It's an equation that includes the square of an unknown. The exponent in these equations is never higher than 2 (because it would then no longer be the *square* of an unknown, but a *cube* or something else). Here are some examples of quadratic equations:

$$x^2 - 4x = -4$$

$$2x^2 = x + 6$$

$$x^2 = 36$$

With most algebra problems, you try to isolate the unknown to solve the problem. Well, not here. When you're solving a quadratic equation, you put all the terms on one side of the equal sign, making the equation equal zero. In other words, get the quadratic equation into this form: $ax^2 + bx + c = 0$.

Look at this equation again:

$$x^2 - 4x = -4$$

To move all the terms to one side, simply add 4 to both sides of the equation:

$$x^2 - 4x + 4 = -4 + 4$$

or

$$x^2 - 4x + 4 = 0$$

Then, use the quadratic formula to solve for x.

To solve a quadratic equation, you use the quadratic formula. The quadratic formula is

$$\frac{x = -b \pm \sqrt{b^2 - 4ac}}{2a}$$

To solve our problem, now you have to plug and chug.

$$x = \frac{4 \pm \sqrt{-4^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 16}}{2}$$

$$x = \frac{4 \pm 0 = 2}{2}$$

SERGEANT SAYS

For the ASVAB, the fastest and easiest way to solve a quadratic equation is to use the trial and error method of substituting each of the four possible answers into the equation that the problem asks you to solve to see which answer is correct.

Solving inequalities

We're talking mathematical inequalities here. Some algebra problems state that two numbers aren't equal to each other (thus they're inequalities). In an inequality, the first number is either greater than or less than the second.

Reviewing rules you need to know

Short and sweet, here they are:

- *Negative numbers* are less than zero and less than positive numbers.
- *Zero* is less than positive numbers but greater than negative numbers.
- *Positive numbers* are greater than negative numbers and greater than zero.

Reviewing symbols you need to know

Certain symbols are used to express inequalities:

- $\neq$ means *does not equal* in the way that 3 *does not equal* 4, or $3 \neq 4$.
- $>$ means *greater than* in the way that 4 *is greater than* 3, or $4 > 3$.
- $<$ means *less than* in the way the 3 *is less than* 4, or $3 < 4$.
- $\leq$ means *less than or equal to* in the way that x may be *less than or equal to* 4, or $x \leq 4$.
- $\geq$ means *greater than or equal to* in the way that x may be *greater than or equal to* 3, or $x \geq 3$.

Getting down business

To solve an inequality, you follow the same rules as you would for solving any other equation. For example, check out this inequality:

$$3 + x \geq 4$$

To solve it, simply isolate the x by subtracting 3 from both sides of the equation:

$$3 + x - 3 \geq 4 - 3$$

or

$$x \geq 1$$

The only exception to this rule is when you multiply or divide both sides of the inequality by a negative number. In that case, the inequality sign is reversed. So, if you multiply both sides of the inequality $3 < 4$ by -4, your answer is $-12 > -16$.

Taking In Some Test-Taking Techniques

As with most of the other subtests on the ASVAB, guessing on the Mathematical Knowledge subtest doesn't count against you. So scribble in an answer, any answer, on your answer sheet because, if you don't, your chances of getting that answer right are 0. But, if you take a shot at it, your chances increase to 25%, or 1 in 4. In the following sections, we lay out some tips that can help you improve those odds, even when you don't know how to solve the problem.

Knowing what the question asks

This subtest presents most of the questions as straightforward math problems, not word problems, so knowing what the question is asking you to do is easier. However, reading each question carefully, paying particular attention to plus and minus signs (which can really change the answer to a question), is still important. Finally, make sure you do *all* of the calculations needed to produce the correct answer. Check out this example:

Find the value of $\sqrt{81^2}$.

(A) 9

(B) 18

(C) 81

(D) 6,561

Correct answer: C. If you're in a hurry, you may put 9 down as an answer because you remember that the square root of 81 is 9. Or, in a rush, you could multiply 9 (the square root of 81) by 2 instead of squaring it, as the exponent indicates you should. Or, you might just multiply 81×81 to get 6,561 without remembering that you also need to find the square root. So make sure you perform all the operations needed (and that you perform the *correct* operations) to find the right answer.

Double-checking on the double

Although you don't have a ton of time to complete the Mathematical Knowledge subtest, you do have about a minute per problem. Although a minute doesn't allow for a lot of head scratching, it's more time than you think. So double-check your answers before putting your pencil down (or before going on to the next problem on the computer).

You can go over your calculations again to make sure that you didn't make an error. You can also plug your answer into the original equation to make sure that it's the correct answer. Then move along, private!

Keep in mind that the questions towards the end of this subtest are harder, so move faster on the early questions to allow for a little more head-scratching time when the ASVAB sends out its big guns.

Guessing your way to better odds

Because the Mathematical Knowledge subtest doesn't penalize you for guessing, mark the answer sheet even if you're clueless. You can even make a pretty design on your answer sheet and still have a one-in-four chance of getting each answer right.

Figuring out what you're solving for

Even though getting artistic with your answer sheet can be fun, we have some techniques that you can try first to improve your chances of guessing the right answer. Right out of the gate, read the question carefully. Some questions can seem out of your league at first glance, but if you look at them again, a light may go on in your brain. Suppose you get this question:

EXAMPLE

s number of students are in a classroom. $\frac{2}{5}$ of the students are enlisted personnel. $\frac{1}{2}$ of the enlisted personnel are privates. How many privates are in the audience?

(A) $2\frac{1}{2}s$

(B) $2s$

(C) $\frac{1}{5}s$

(D) $\frac{1}{10}s$

At first glance, you may think, "Oh, no! Solve for an unknown, *s*. I don't remember how to do that!" But, if you look at the question again, you may see that you're not solving for *s* at all. You're simply multiplying a fraction. So you take $\frac{2}{5}$ times $\frac{1}{2}$ and arrive at $\frac{1}{5}$. *Correct answer:* C. (See Chapter 6 for a refresher on multiplying fractions.)

Solving what you can and guessing the rest

Sometimes a problem requires multiple operations for you to arrive at the correct answer. If you don't know how to do all of the operations, don't give up. You can still narrow your guess down by doing what you can. Suppose this question confronts you:

EXAMPLE

What is the value of $(0.03)^3$?

(A) 0.0027

(B) 0.06

(C) 0.000027

(D) 0.0009

Say you don't remember how to multiply decimals. All is not lost! If you remember how to use exponents, you'll remember that you have to multiply $0.03 \times 0.03 \times 0.03$. So, if you simplify the problem and just multiply $3 \times 3 \times 3$, without worrying about those pesky zeroes, you'll know that your answer will have a *27* in it. With this pearl of wisdom in mind, you can see that Answer B, which is arrived at by adding 0.03 to 0.03, is wrong. It also means that Answer D, which is reached by multiplying 0.03×0.03, is wrong. Now you have two possible answers, and you've improved your chances of guessing the right one to 50 percent! *Correct answer:* C.

SERGEANT SAYS

Don't forget to use that scratch paper! Suppose you run across this question: "A child is building a tower of blocks. Each block is a cube. Some blocks are white, and some blocks are red. Red blocks surround each white block. How many red blocks surround each white block?" This problem may be difficult to figure out, until you sketch a six-sided block (a cube) on your scratch paper and realize that the block must be surrounded by six other blocks.

Using the answers to find the answer

Say the following problem is staring you right in the eyes:

Solve for x: $x - 5 = 32$.

(A) $x = 5$

(B) $x = 32$

(C) $x = -32$

(D) $x = 37$

You're not sure what to do. If you're totally stumped and can't think of any possible way of approaching this problem, simply plugging in each of the four answers to see which one is correct is your best bet.

- **Answer A:** $5 - 5 = 32$, which you know is wrong.
- **Answer B:** $32 - 5 = 32$, which is wrong.
- **Answer C:** $-32 - 5 = 32$, which is wrong.
- **Answer D:** $37 - 5 = 32$, which is correct.

Don't forget that plugging in all the answers is time consuming, so save this procedure until you've answered all the problems you can answer. If you're taking the computer version, you can't skip a question, so remember to budget your time wisely. If you don't have much time, just make a guess and move on. You may be able to solve the next question easily.

Sample Questions

Now that you've honed your mathematical skills, take them out for a test drive with these questions.

1. Which of the following fractions is the largest?

(A) $\frac{2}{3}$

(B) $\frac{5}{8}$

(C) $\frac{11}{16}$

(D) $\frac{3}{4}$

Correct answer: D. To arrive at this answer, find a common denominator that all the denominators divide evenly into. In this case, the common denominator is 48 (discovered by multiplying 16×3). Next, convert all fractions to 48ths. In the case of Choice A, multiply $\frac{2}{3} \times \frac{16}{16}$ to reach $\frac{32}{48}$. Perform the same type of calculation for all the other fractions and then compare numerators. The largest numerator is the largest fraction.

2. What is the product of $\sqrt{36}$ and $\sqrt{49}$?

(A) 1,764

(B) 42

(C) 13

(D) 6

Correct answer: B. The square root of 36 is 6 and the square root of 49 is 7. The product of those two numbers (6×7) is 42.

3. Solve for x: $2x - 3 = x + 7$.

   (A) 10

   (B) 6

   (C) 21

   (D) –10

   Correct answer: A. Isolate the x's on one side of the equation by subtracting x from both sides: $2x - 3 - x = x + 7 - x$, or $x - 3 = 7$. Continue to perform operations to isolate x. Add 3 to both sides of the equation: $x - 3 + 3 = 7 + 3$, or $x = 10$.

4. A circle has a radius of 15 feet. What is most nearly its circumference?

   (A) 30 feet

   (B) 225 feet

   (C) 94 feet

   (D) 150 feet

   Correct answer: C. The circumference of a circle is pi × diameter; the diameter equals two times the radius. Therefore $30 \times 3.14 = 94$.

5. At 3:00 p.m., the angle between the hands of the clock is:

   (A) 90 degrees

   (B) 180 degrees

   (C) 120 degrees

   (D) 360 degrees

   Correct answer: A. At 3 p.m., one hand is on the 12, and the other is on the 3. This creates a right angle — a 90-degree angle.

MATHEMATICS KNOWLEDGE REVIEW

The Mathematics Knowledge subtest of the ASVAB deals with the ability to use basic mathematical relationships learned in math courses, such as algebra, geometry, and trigonometry. This subtest tests your knowledge of math principles, concepts, and procedures. This review section can also be considered a continuation of the Arithmetic Reasoning Review given previously. Reread that section as background material for Mathematics Knowledge Review.

Adding Fractions

With the Same Denominator

Fractions with the same denominator are added directly, as each part represents a part of the same value. Add the numerators and place the sum over the common denominator. If necessary, simplify to the simplest form.

$$\begin{array}{r} \frac{1}{5} \\ +\frac{3}{5} \\ \hline \frac{4}{5} \end{array} \qquad \begin{array}{r} \frac{1}{5} \\ \frac{3}{5} \\ +\frac{4}{5} \\ \hline \frac{8}{5} = 1\frac{3}{5} \end{array}$$

With Different Denominators

Fractions with different denominators may not be added directly because parts of different values are involved. They must be renamed as equivalent fractions having the same common denominator. After all the fractions have the same denominator, add the numerators and place the total over the common denominator. If necessary, simplify to the simplest form.

$$\frac{1}{2}+\frac{1}{4};\ \frac{1}{2}+\frac{1}{3}+\frac{3}{4}$$

$$\begin{array}{r} \frac{1}{2} \\ +\frac{1}{4} \\ \hline \end{array} \qquad \begin{array}{r} \frac{2}{4} \\ +\frac{1}{4} \\ \hline \frac{3}{4} \end{array} \qquad \begin{array}{r} \frac{1}{2} \\ \frac{1}{3} \\ +\frac{3}{4} \\ \hline \end{array} \qquad \begin{array}{r} \frac{6}{12} \\ \frac{4}{12} \\ +\frac{9}{12} \\ \hline \frac{19}{12}=1\frac{7}{12} \end{array}$$

Adding Mixed Numbers

In adding mixed numbers, first add all the whole numbers, then add all fractions, and then add the sum of the whole numbers to the sum of the fractions.

$$4\frac{1}{4}+3\frac{1}{2}+2\frac{1}{2}$$

$$\begin{array}{r} 4\frac{1}{4} \\ 3\frac{1}{2} \\ +2\frac{1}{2} \\ \hline \end{array} \qquad \begin{array}{r} 4\frac{1}{4} \\ 3\frac{2}{4} \\ +2\frac{2}{4} \\ \hline \end{array}$$

$$9+\frac{5}{4}=9+1\frac{1}{4}=10\frac{1}{4}$$

Adding Percents

As percents are actually fractions with 100 as the same common denominator, they may be added directly.

6%		20%
4%	8%	25%
+9%	+17%	+35%
19%	25%	80%

If fractional parts of a percent are involved in the addition, the addends may be added as decimals or added directly after the fractional parts are renamed with the same common denominator.

$$15\frac{1}{2}\% + 8\frac{1}{4}\%$$

$$\begin{array}{r} 0.155 \\ +0.0825 \\ \hline 0.2375 \end{array}$$

$$\begin{array}{rr} 15\frac{1}{2}\% & 15\frac{2}{4}\% \\ +8\frac{1}{4}\% & +8\frac{1}{4}\% \\ \hline & 23\frac{3}{4}\% \end{array}$$

Subtracting Fractions

With the Same Denominator

Fractions with the same denominator may be subtracted directly.

$\frac{3}{5} - \frac{2}{5} = \frac{1}{5}$ $\quad \frac{7}{8} - \frac{1}{8} = \frac{6}{8}$, which simplifies to $\frac{3}{4}$.

With Different Denominators

Fractions with different denominators are not subtracted directly because parts of different values are involved. They must be renamed as equivalent fractions having the same common denominator. After the fractions have the same denominator, subtract the numerators and place the remainder over the common denominator.

$$\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12} \qquad \frac{4}{5} - \frac{1}{2} = \frac{8}{10} - \frac{5}{10} = \frac{3}{10}$$

Subtracting Mixed Numbers

With the Same Denominator

If mixed numbers have the same denominator, rename them as improper fractions and subtract directly.

$$4\frac{1}{3} - 2\frac{2}{3} = \frac{13}{3} - \frac{8}{3} = \frac{5}{3} = 1\frac{2}{3}$$

With Different Denominators

If mixed numbers have different denominators, first rename as improper fractions, then rename as equivalent fractions with the same common denominator and subtract directly.

$$3\frac{1}{3}-2\frac{3}{4}=\frac{10}{3}-\frac{11}{4}=\frac{40}{12}-\frac{33}{12}=\frac{7}{12}$$

Subtracting Percents

Percents are fractions with 100 as the same common denominator and may be subtracted directly.

$$70\% \text{ minus } 30\% = 40\%$$

If fractional parts of a percent are involved in the subtraction, rename them as decimals or rename the fractional parts with the same common denominator.

$$8\frac{1}{4}\%-5\frac{2}{5}\%$$

$$\begin{array}{rrrr} & 8\frac{1}{4}\% & 8\frac{5}{20}\% & 7\frac{25}{20}\% \\ 0.0825 & & & \\ -\ 0.0540 & -\ 5\frac{2}{5}\% & -\ 5\frac{8}{20}\% & -\ 5\frac{8}{20}\% \\ \hline 0.0285 & & & 2\frac{17}{20}\% \end{array}$$

Multiplying Fractions

With fractions, the product of the numerators divided by the product of the denominators gives the final answer or product.

$$\frac{1}{2}\times\frac{2}{3}=\frac{2}{6}, \text{ which simplifies to } \frac{1}{3}.$$

Dividing a number in the numerator by the same number in the denominator simplifies the computation.

$$\frac{1}{\underset{1}{\not 2}}\times\frac{\overset{1}{\not 2}}{3}=\frac{1}{3}$$

Dividing a common factor is particularly useful when multiplying many fractions.

$$\frac{3}{5}\times\frac{1}{2}\times\frac{2}{3}\times\frac{5}{8}=\frac{3\times1\times2\times5}{5\times2\times3\times8}=\frac{30}{240}=\frac{1}{8}$$

With dividing common factors:

$$\frac{3}{5}\times\frac{1}{2}\times\frac{2}{3}\times\frac{5}{8}=\frac{\overset{1}{\cancel{3}}\times1\times\overset{1}{\cancel{2}}\times\overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}}\times\underset{1}{\cancel{2}}\times\underset{1}{\cancel{3}}\times8}=\frac{1}{8}$$

Note that dividing common factors is permitted when only multiplication is involved.

When multiplying fractions and whole numbers, use the same procedure as when multiplying fractions only. Whole numbers are basically fractions with the whole number as the numerator and one as the denominator.

$$4=\frac{4}{1} \qquad 10=\frac{10}{1} \qquad 150=\frac{150}{1}$$

When multiplying a fraction and a whole number, the word *of* means *multiply* by.

$\frac{1}{2}$ of 48 means $\frac{1}{2}\times\frac{48}{1}$, which equals $\frac{48}{2}$ and equals 24.

Multiplying Mixed Numbers

There are several methods that may be used in multiplying mixed numbers.

When the numbers are small valued, rename the mixed numbers as improper fractions and then multiply in the usual manner.

$$3\frac{1}{4}\times16=\frac{13}{4}\times\frac{16}{1}=\frac{208}{4}=52$$

$$2\frac{2}{3}\times1\frac{1}{4}=\frac{8}{3}\times\frac{5}{4}=\frac{40}{12}=\frac{10}{3}=3\frac{1}{3}$$

$$1\frac{1}{2}\times2\frac{2}{3}\times3\frac{3}{4}=\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}}\times\frac{\overset{1}{\cancel{8}}}{\underset{1}{\cancel{3}}}\times\frac{15}{\underset{1}{\cancel{4}}}=15$$

When the numbers are great and the fractional parts have exact decimal equivalents, rename the fractional parts as decimals and then multiply.

$$342\frac{1}{4} \times 609\frac{3}{4} = 342.25 \times 609.75$$

$$\begin{array}{r} 342.25 \\ \underline{\times 609.75} \\ 171125 \\ 239575 \\ 308025 \\ \underline{2053500} \\ 208,686.9375 \end{array}$$

When the numbers are not small valued and the fractional parts have no exact decimal equivalents, use the partial product method as follows:

$$386\frac{3}{7} \times 245\frac{1}{3}$$

$$\begin{array}{rl} 386\frac{3}{7} & \\ \underline{\times 245\frac{1}{3}} & \\ \frac{3}{21} & \left(\frac{1}{3} \times \frac{3}{7}\right) \\ 128\frac{2}{3} & \left(\frac{1}{3} \times 386\right) \\ 105 & \left(\frac{3}{7} \times 245\right) \\ \underline{94,570} & (245 \times 386) \\ 94,803\frac{17}{21} & \end{array}$$

Find the partial products of the

a. fractional parts of the multiplier and the multiplicand.
b. fractional part of the multiplier and the whole number of the multiplicand.
c. fractional part of the multiplicand and the whole number of the multiplier.
d. whole number part of the multiplier and the whole number part of the multiplicand.

Add the partial products and, if necessary, simplify the fractional part of the answer.

Multiplying Percents

Since a percent is actually a fraction with a denominator of 100, it may be multiplied after renaming the percent as a decimal or as a fraction.

$$33\% \times 8\% = .33 \times .08 = .0264 = 2\frac{64}{100}\% = 2\frac{32}{50}\% = 2\frac{16}{25}\%$$

$$75\% \times 50\% = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8} = 37\frac{1}{2}\%$$

Other Multiplication Properties

1. If the multiplier and the multiplicand are interchanged, the product will remain the same.

   $5 \times 4 = 20 \qquad 4 \times 5 = 20$

2. If the numbers being multiplied are associated in different ways, the product will remain the same.

   $3 \times (5 \times 2) = 2 \times (3 \times 5) = 5 \times (2 \times 3)$

3. Multiplying any number by 1 does not change the number.

   $8 \times 1 = 8 \qquad 1.5 \times 1 = 1.5 \qquad \frac{3}{4} \times 1 = \frac{3}{4}$

4. Zero times any number equals zero.

   $8 \times 0 = 0 \qquad 1.5 \times 0 = 0 \qquad \frac{3}{4} \times 0 = 0$

5. a. To multiply by 10, move the decimal point in the number one place to the right:

   $1.36 \times 10 = 13.6,\ 13.6 \times 10 = 136,\ 136 \times 10 = 1{,}360$

   b. To multiply by 100, move the decimal point in the number two places to the right.

   $3.61 \times 100 = 361,\ 36.1 \times 100 = 3{,}610,\ 361 \times 100 = 36{,}100$

   c. To multiply by 1,000, move the decimal point in the number three places to the right.

$4.875 \times 1{,}000 = 4{,}875$
$48.75 \times 1{,}000 = 48{,}750$
$487.5 \times 1{,}000 = 487{,}500$
$4{,}875 \times 1{,}000 = 4{,}875{,}000$

When multiplying by 10, 100, 1,000, etc., move the decimal point in the number as many places to the right as there are zeros in the multiplier. If necessary, add zero(s) to the product.

Dividing Fractions

With fractions, multiply by the reciprocal of the divisor.

$\frac{3}{4} \div \frac{1}{2}$ is rewritten as $\frac{3}{4} \times \frac{2}{1}$, which equals $\frac{6}{4} = 1\frac{1}{2}$
$\frac{2}{3} \div \frac{3}{4}$ is rewritten as $\frac{2}{3} \times \frac{4}{3}$, which equals $\frac{8}{9}$
$3 \div \frac{3}{4}$ is rewritten as $\frac{3}{1} \times \frac{4}{3}$, which equals $\frac{12}{3} = 4$
$\frac{1}{4} \div 2$ is rewritten as $\frac{1}{4} \times \frac{1}{2}$, which equals $\frac{1}{8}$

Dividing Percents

A percent divided by a percent is similar to dividing two fractions with 100 as the common denominator.

$25\% \div 50\%$ is rewritten as $\frac{25}{100} \times \frac{100}{50}$, which equals $\frac{25}{50} = \frac{1}{2}$

$40\% \div 40\%$ is rewritten as $\frac{40}{100} \times \frac{100}{40}$, which equals 1

Note that when a percent is divided by a percent, the result is a whole number or a fraction.

$\frac{1}{2} \div 25\%$ is rewritten as $\frac{1}{2} \times \frac{100}{25}$, which equals $\frac{100}{50} = 2$
$25\% \div \frac{1}{2}$ is rewritten as $\frac{25}{100} \times \frac{2}{1}$, which equals $\frac{50}{100} = \frac{1}{2}$

Similarly, when a fraction is divided by a percent or a percent is divided by a fraction, the result is also a whole number or a fraction.

Percents may also be renamed to decimal form before division.

$$25\% \div 50\% \text{ is rewritten as } \frac{.25}{.50}, \text{ which equals } \frac{25}{50} = \frac{1}{2}$$

$$\frac{1}{2} \div 25\% \text{ is rewritten as } \frac{1}{2} \div .25 = \frac{1}{2} \times \frac{1}{.25} = \frac{1}{.50} = \frac{100}{50} = 2$$

$$25\% \div \frac{1}{2} \text{ is rewritten as } .25 \div \frac{1}{2} = .25 \times 2 = .50 = \frac{1}{2}$$

Dividing Mixed Numbers

There are several methods that are used to divide mixed numbers.

When the numbers are small valued, rename the mixed numbers as improper fractions and then divide in the usual manner.

$$2\frac{1}{4} \div 1\frac{1}{2} \text{ is rewritten as } \frac{9}{4} \div \frac{3}{2}, \text{ which becomes } \frac{9}{4} \times \frac{2}{3} \text{ and equals } \frac{3}{2} = 1\frac{1}{2}$$

$$1\frac{1}{2} \div 2\frac{1}{4} \text{ is rewritten as } \frac{3}{2} \div \frac{9}{4}, \text{ which becomes } \frac{3}{2} \times \frac{4}{9} \text{ and equals } \frac{2}{3}$$

When the mixed numbers are great and the fractional parts have exact decimal equivalents, rename the fractional parts as decimals and then divide.

$$432\frac{3}{5} \text{ divided by } 156\frac{1}{2} \text{ is rewritten as } 432.6 \div 156.5$$

$$1565\overline{)4326}$$

When the mixed numbers are great and the fractional parts do not have exact decimal equivalents,

1. Multiply both the dividend and the divisor by the denominator of the fraction if only one fraction is involved:

$$475 \div 28\frac{2}{3} = \frac{475 \times 3}{28\frac{2}{3} \times 3} = \frac{475 \times 3}{\frac{86}{3} \times 3} = \frac{1425}{86}$$

$$27\frac{1}{6} \div 39 = \frac{27\frac{1}{6} \times 6}{39 \times 6} = \frac{\frac{163}{6} \times 6}{39 \times 6} = \frac{163}{234}$$

2. Multiply both the dividend and the divisor by the least common denominator if two fractions are involved:

$$42\frac{2}{3} \div 12\frac{1}{6} = \frac{42\frac{2}{3} \times 6}{12\frac{1}{6} \times 6} = \frac{\frac{128}{3} \times 6}{\frac{73}{6} \times 6} = \frac{256}{73}$$

Other Division Properties

1. If the division is not exact, the number that is left is the remainder. This remainder becomes the numerator, and the divisor becomes the denominator of this common fraction that is added to the quotient:

$$4\overline{)1207}$$

$$301\frac{3}{4}$$

2. The remainder may also be shown as a decimal. The decimal point is placed to the right of the unit digit of the dividend and zero digits are added. Divide to the desired number of decimal places:

$$4\overline{)1207}.00$$

$$301.75$$

3. Dividing any number by 1 does not change the number:

$$25 \div 1 = 25 \qquad 4.7 \div 1 = 4.7 \qquad \frac{3}{5} \div 1 = \frac{3}{5}$$

4. Dividing by zero is not permissible, because the answer indicated would be undefined.

5. a. To divide by ten, move the decimal point in the number one place to the left:

$$\frac{136}{10} = 13.6 \qquad \frac{13.6}{10} = 1.36 \qquad \frac{1.36}{10} = .136$$

b. To divide by 100, move the decimal point in the number two places to the left:

$$\frac{350}{100} = 3.50 \qquad \frac{35}{100} = 0.35 \qquad \frac{3.5}{100} = .035$$

c. To divide by 1000, move the decimal point in the number three places to the left:

$$\frac{4055}{1000} = 4.055 \quad \frac{40.55}{1000} = .04055$$
$$\frac{405.5}{1000} = .4055 \quad \frac{4.055}{1000} = .004055$$

When dividing by 10, 100, 1000, etc., move the decimal point in the number as many places to the left as there are zeros in the divisor. If necessary, use zeros at the left of the dividend.

Factors of a Product

When two or more numbers are multiplied to produce a certain product, each number is known as a *factor* of the product.

$1 \times 8 = 8$ (1 and 8 are factors of the product)

$2 \times 4 = 8$ (2 and 4 are factors of the product)

Base

A *base* is a number used as a factor two or more times. $2 \times 2 \times 2$ may be written 2^3, which is read "2 cubed" or "2 to the third power." In the equation $2^3 = 8$, 2 is called the base.

Exponent

The *exponent* is the number that shows how many times the base is to be used as a factor. 10^2 is a short way of writing 10 ′ 10. 10 is called the base in 10^2; 2 is called the exponent.

$a^4 = a \times a \times a \times a$ (*a* is the base, 4 is the exponent)

$5^3 = 5 \times 5 \times 5$ (5 is the base, 3 is the exponent)

Power

Power is an expression such as 3^2. 3^2 is the second power of three (3 ′ 3) and is equal to 9. 2^4 is the fourth power of two ($2 \times 2 \times 2 \times 2$) and is equal to 16. Note that all the factors of the product are equal.

Reciprocal

If the product of two numbers is 1, either number is called the *reciprocal* of the other number. 4 is the reciprocal of $\frac{1}{4}$; $\frac{1}{4}$ is the reciprocal of 4; $4 \times \frac{1}{4} = 1$. Similarly, $\frac{3}{5}$ is the reciprocal of $\frac{5}{3}$; $\frac{5}{3}$ is the reciprocal of $\frac{3}{5}$; $\frac{3}{5} \times \frac{5}{3} = 1$.

Factorial

The *factorial* of a natural or counting number is the product of that number and all the natural numbers less than it. **4 factorial**, written as $4! = 4 \times 3 \times 2 \times 1 = 24$.

Prime Number

A *prime number* is a natural or counting number with exactly two factors, namely itself and 1. Examples of prime numbers are 2, 3, 5, 7, 11, 13, 17, etc.

Roots

Square Root

The *square root* of a number is a number that, when raised to the second power, produces the given number. For example, the square root of 16 is 4 because $4^2 = 16$. $\sqrt{\ }$ is the symbol for square root.

The square roots of the most common perfect squares are given in the following table below.

NUMBER	PERFECT SQUARE	NUMBER	PERFECT SQUARE
1	1	10	100
2	4	11	121
3	9	12	144
4	16	13	169
5	25	14	196
6	36	15	225
7	49	20	400
8	64	25	625
9	81	30	900

For example, to find $\sqrt{81}$, note that 81 is the perfect square of 9, or $9^2 = 81$. Therefore, $\sqrt{81} = 9$.

Cube Root

Cube root is the procedural inverse of raising to a cube. If $2^3 = 8$, then $\sqrt[3]{8} = 2$. The cube root of $27 = 3$; $3^3 = 27$.

Algebra

Algebra is the branch of mathematics that focuses on addition, subtraction, multiplication, and division operations applied to variables, or unknowns, instead of specific numbers.

Here are these operations in algebraic form:

Operation	Algebraic Form
Addition: The sum of two numbers	$x + y$
Subtraction: The difference between two numbers	$x - y$
Multiplication: The product of two numbers	$x \times y$ *or* xy
Division: The quotient of two numbers	$\frac{x}{y}$

Algebraic Equations

An *equation* states that two quantities are equal. The solution to an equation is a number that can be substituted for the letter, or *variable*, to give a true statement.

For example, in the equation $x + 7 = 10$, if 5 is substituted for x, the equation becomes $5 + 7 = 10$, which is false. If 3 is substituted for x, the equation becomes $3 + 7 = 10$, which is true. Therefore, $x = 3$ is a solution for the equation $x + 7 = 10$.

An equation has been solved when it is transformed or rearranged so that a variable or unknown is on one side of the equal sign and a number is on the other side.

There are two basic principles that are used to transform equations:

1. The same quantity may be added to, or subtracted from, both sides of an equation.

   To solve the equation $x - 3 = 2$, add 3 to both sides:

$$\begin{array}{rcl} x - 3 &=& 2 \\ \underline{+3} & & \underline{+3} \\ x &=& 5 \end{array}$$

   Adding 3 isolates x on one side and leaves a number on the other side. The solution to the equation is $x = 5$.

To solve the equation $y + 4 = 10$, subtract 4 from both sides (adding –4 to both sides will have the same effect):

$$\begin{array}{rcl} y+4 & = & 10 \\ \underline{-4} & & \underline{-4} \\ y & = & 6 \end{array}$$

The variable has been isolated on one side of the equation. The solution is $y = 6$.

2. Both sides of an equation may be multiplied by, or divided by, the same quantity.

To solve $2a = 12$, divide both sides by 2:

$$\frac{2a}{2} = \frac{12}{2}$$
$$a = 6$$

To solve $\frac{b}{5} = 10$, multiply both sides by 5:

$$5 \cdot \frac{b}{5} = 10 \cdot 5$$
$$b = 50$$

To solve equations containing more than one operation:

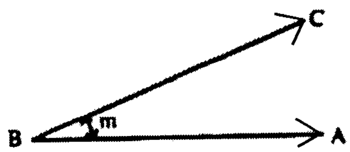
First, eliminate any number that is being added to or subtracted from the variable. Then eliminate any number that is multiplying or dividing the variable (A number that is multiplying the variable is called a *coefficient.*)

Solve:	$3x - 6$	=	9	
	$\underline{+6}$		$\underline{+6}$	Adding 6 eliminates -6.
	$3x$	=	15	
	$\frac{3x}{3}$	=	$\frac{15}{3}$	Dividing by 3 eliminates the 3 that is multiplied by the x.
	x	=	5	The solution to the original equation is $x = 5$.

Geometry

Angles

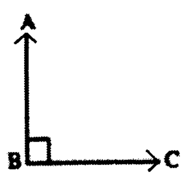
An *angle* is the figure formed by two rays meeting at a point.



The point B is the *vertex* of the angle, and $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are the *sides* of the angle. The symbol for an angle is ∠. The letter *m* stands for the measure of the angle in degrees.

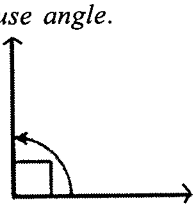
Types of Angles

1. When two straight lines intersect (cut each other), four angles are formed. If these four angles are equal, each angle is a *right angle* and contains 90°. The symbol is used to indicate a right angle, as shown below.

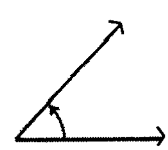


∠ABC is a right angle.

2. An angle that is smaller than a right angle is an *acute angle.*
3. If the two sides of an angle extend in opposite directions forming a straight line, the angle is a *straight angle* and measures 180°.
4. An angle that is bigger than a right angle (90°) and smaller than a straight angle (180°) is an *obtuse angle.*

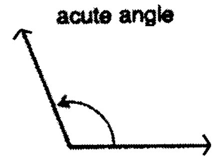


right angle



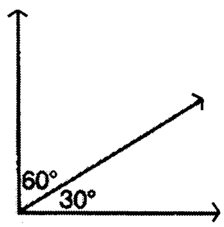
acute angle

straight angle

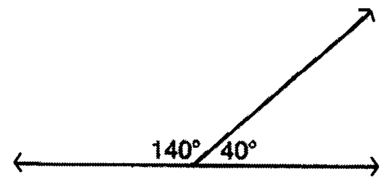


obtuse angle

Complementary angles are two angles whose measures sum to 90°. Each angle is the complement of the other. If an angle measures 30°, its complement measures 60°. If an angle measures $x°$, its complement measures $(90 - x)°$.



Complementary Angles

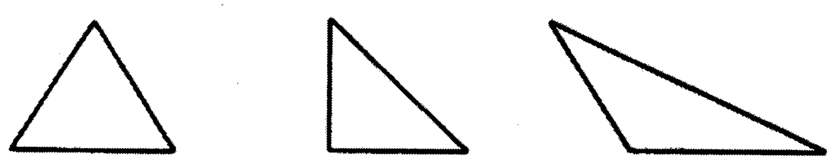


Supplementary Angles

Supplementary angles are two angles whose measures sum to 180°. Each angle is the supplement of the other. If an angle measures 140°, its supplement measures 40°. If an angle measures $x°$, its supplement measures $(180 - x)°$.

Triangles

A *triangle* is a closed, three-sided figure. The following figures are triangles.



The sum of the measures of three angles of a triangle is 180°.

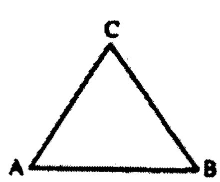
To find the measure of an angle of a triangle given the measure of the other two angles, add the measures and subtract their sum from 180°.

For example, if the measures of two angles of a triangle are 60° and 40°, the measure of the third angle is

$$180° - (60° + 40°) =$$
$$180° - 100° = 80°$$

A triangle with two congruent sides is called an *isosceles triangle.*

In an isosceles triangle, the angles opposite the congruent sides are also congruent.



If AC = BC, then $m\angle A = m\angle B$

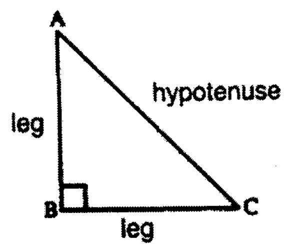
A triangle with all three sides congruent is called an *equilateral triangle.*

Each angle of an equilateral triangle measures 60°.

A triangle with a right angle is called a *right triangle.*

In a right triangle, the two acute angles are complementary.

In a right triangle, the side opposite *the right angle* is called the *hypotenuse* and is the longest side. The other two sides are called *legs.*



In right triangle ABC, $\overline{AC}$ is the hypotenuse. $\overline{AB}$ and $\overline{BC}$ are the legs.

The *Pythagorean theorem* states that in a right triangle, the square of the hypotenuse equals the sum of the squares of the legs.

In right triangle ABC: $(AC)^2 = (AB)^2 + (BC)^2$

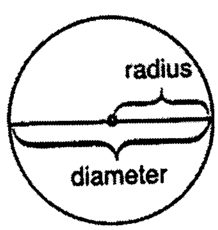
Circles

A *circle* is a closed plane curve, all points of which are equidistant from a point within called the center.

A complete circle contains 360°.

A *radius* of a circle is a line segment connecting the center with any point on the circle.

A *diameter* of a circle is a line segment connecting any two points on the circle and passing through the center of the circle. The diameter of any circle is twice the radius of that circle.



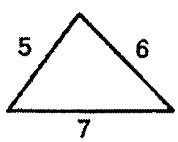
Perimeter

The *perimeter* of a two-dimensional figure is the distance around the figure. The perimeter of a rectangle equals twice the sum of the length and the width.

$$P = 2(l + w)$$
$$P = 2(8 + 4) = 2(12)$$
$$P = 24$$

The perimeter of a triangle is the sum of the three sides.

$$P = 7 + 6 + 5 = 18$$

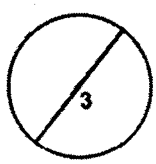


The perimeter of a circle is called the circumference. The circumference of a circle is equal to the product of the diameter multiplied by π.

The formula is $C = \pi d$

Pi (π) is a mathematical value equal to approximately 3.14 or $\frac{22}{7}$.

$$C = \pi d$$
$$C = \pi(3) = 3\pi$$



Area

In a two-dimensional figure, the total space within the figure is called the *area.*

Area is expressed in square denominations, such as square inches, square centimeters, or square miles.

The area of a rectangle equals the product of the length (or base) multiplied by the width (or height).

$A = lw$

$A = 9\ ft. \times 3\ ft.$

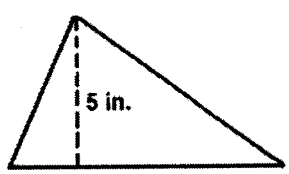
$A = 27\ sq.\,ft.$

The area of a triangle is equal to one half the product of the base and the height. The height (or altitude) of a triangle is a line drawn from a vertical perpendicular to the opposite side, called the base.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(9 \text{ in.})(5 \text{ in.}) = \frac{45}{2}$$

$$A = 22\frac{1}{2} \text{sq. in.}$$

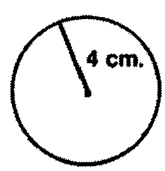


The area of a circle is equal to the radius squared multiplied by π.

$A = \pi r^2$

$A = \pi(4 \text{ cm.})^2$

$A = 16\pi \text{ sq. cm.}$



For some ASVAB questions, you may leave the area in terms of pi.